Seismic behaviour assessment of columns made up of rigid blocks

Luigi Petti[a], Domenico Greco[a], Filomena Nuccio[a]
[a] *Università degli Studi di Salerno, Dipartimento di Ingegneria Civile, Via Via Ponte Don Melillo, 84084 Fisciano (Sa), Italy*

Keywords: Rigid body dynamics; Column preservation; Seismic response.

ABSTRACT
Ancient architectures made up of stone blocks present an intrinsic appeal for both artistic and engineering point of view. The preservation of such monuments, especially in seismic prone area, even now, represents a challenge for civil engineering.
The paper presents a wide experimental campaign aimed to investigate the behaviour of columns made up of rigid blocks. The experimental results are compared with classical theoretical formulations to highlight the feasibility of simplified analysis methods in the case of large seismic demand. In particular, the carried-out tests lead to implement simplified equivalent models useful to investigate the seismic capacity of these particular structures. The obtained results, confirmed by numerical analysis carried out by the Distinct Element Method (DEM), lead to evaluate the seismic safety of sample structures such as the Greek Neptune Temple in Paestum (Southern Italy).

1 INTRODUCTION

One of the most critical issues in the field of conservation and protection of Cultural Heritage in earthquake-prone areas is represented by the study of the dynamic behaviour of monumental buildings constituted by rigid blocks, such as the Greek temples (UNESCO, 2013) as the Temple of Neptune in Paestum (Italy) (Figure 1).

Among the first studies on the dynamic behaviour of a rigid body, the researches of Housner can be found (Housner, 1963).

The inverted pendulum model of Housner (Figure 2) defines the motion of a non-deformable body in rigid contact interface conditions. Nowadays, dissimilar approaches are available in the literature considering different contact types at the interface between the blocks: rigid contact models and elastic contact models can be distinguished (Giannini, 1991).

Other recent studies have shown the impossibility of modeling the phenomenon of rocking in multi-block columns with a single rigid system (Minafò and Amato, 2016). Some research studies on the seismic response of this type of structures were also carried out (Konstantinidis and Makris, 2005), with the aim of determining the level of seismic shaking in conditions of incipient collapse. Moreover, in the literature, there are experimental tests on shaking table of a portal made by two columns formed by different rigid blocks (Drosos and Anastasopoulos, 2014) and release tests to assess main dynamic parameters (Petti, Sicignano and Greco, 2018).

Figure 1 - Neptune temple of Paestum, Southern Italy.

2 RIGID BODY DINAMIC BEHAVIOUR

The model of Housner is represented by a homogeneous parallelepiped block on rigid ground with no sliding at the base: the motion is described by a single Lagrangian coordinate, represented by the rotation θ.

In the described model (Figure 2), G represents the centre of gravity, R is the semi-diagonal, h the semi-height, b the semi-base, m the overall mass and $W=mg$ the weight of the block. Significant quantities which describe the geometry of the block are thus two, the semi-diagonal R, which expresses the size of the block, and the angle α, which provides a block slenderness ratio. Also, θ is the rotation with which the block oscillates alternately around its two lower vertices O and O'.

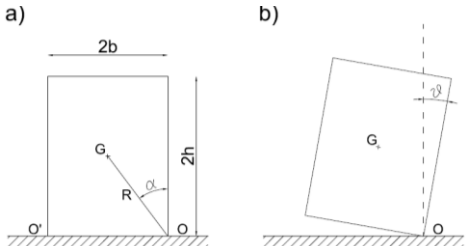

Figure 2: Dynamic behaviour of rigid body.

The motion of the system can be expressed by the Cardinal Equations of dynamics:

$$\frac{\mathrm{d}\boldsymbol{U}}{\mathrm{dt}} = \boldsymbol{R}^{(e)}$$
$$\frac{\mathrm{d}\boldsymbol{K}}{\mathrm{dt}} + \boldsymbol{v_0} \times \mathbf{U} = \boldsymbol{M}^{(e)} \tag{1}$$

where $\boldsymbol{U}$ is the resultant of the quantity of motion, $\boldsymbol{K}$ is the moment of the quantity of motion with respect to the pole O, characterized by a velocity motion $\boldsymbol{v_0}$, and finally $\boldsymbol{R}^{(e)}$ and $\boldsymbol{M}^{(e)}$ are respectively the resultant of external forces and the resulting moment.

Relations (1), in the described hypotheses, lead to the following equilibrium of motion of the rigid block under free oscillations and, in the case of small rotations and slender blocks ($\alpha \leq 20°$) this equation is simplified as follows:

$$\ddot{\theta} = -p^2 \cdot \alpha + p^2 \cdot \theta \tag{2}$$

since p is the frequency parameter and I_O the mass moment of inertia concerning the pole O:

$$p = \sqrt{\frac{mgR}{I_0}} \qquad I_0 = \frac{4}{3} m \cdot R^2$$

In the hypothesis of null initial angular velocity, the relation (2) allows to evaluate the natural vibration period as follows:

$$T = \frac{4}{p} \cosh^{-1}\left(\frac{1}{1-\theta_0/\alpha}\right) \tag{3}$$

This relation shows that, unlike the linear case, the oscillation period depends on the initial rotation. Figure 3 describes the effects on the response due to the linearization of the equation (2) by comparing the dimensionless restoring moments related to both the simplified and the exact formulation. The comparison shows that the linearization of dynamic equations leads to a slight error for the slender ratio $\alpha<20°$; more significant errors are instead obtained for $\alpha>60°$, which correspond to a slender ratio of $\theta/\alpha<0.3$.

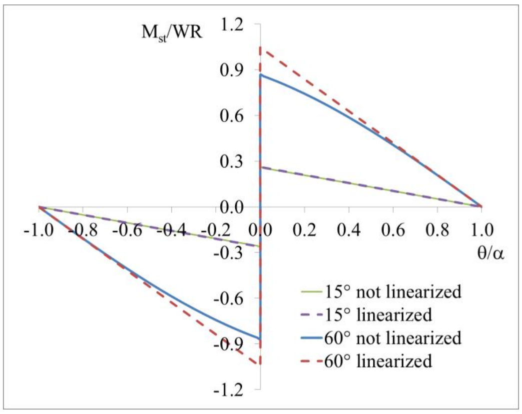

Figure 3: Dimensionless moment-rotation domain for different initial rotation angle values (15° and 60°).

Furthermore, the energy dissipation due to the collisions at the base can be described using the restitution coefficient (e), which depends on the kinetic energy variation (r). (Petti, Sicignano, Greco, ANIDIS 2017; Petti, Sicignano, Greco, 16ECEE 2018).

$$r = \frac{\frac{1}{2} \cdot I_0 \cdot \dot{\theta}_2^{\,2}}{\frac{1}{2} \cdot I_0 \cdot \dot{\theta}_1^{\,2}} = \left(\frac{\dot{\theta}_2}{\dot{\theta}_1}\right)^2 = e^2 \tag{4}$$

The subscripts (1) and (2) respectively represent the time immediately before and after the collision. By setting the angular momentum to the rotation pole O, it is then possible to evaluate the restitution coefficient as follows:

$$I_O \cdot \dot{\theta}_1 - 2 \cdot m \cdot R \cdot b \cdot \dot{\theta}_1 \cdot \mathrm{sen}\,(\alpha) = I_O \cdot \dot{\theta}_2$$

$$e = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \sqrt{r} = \frac{1}{4} + \frac{3}{4} \cdot \cos(2\alpha) \tag{5}$$

The restitution coefficient can be assessed by experimental data, observing that the kinetic energy variation to the impact has to be equal to the potential energy variation of the block in the maximum rotation configurations; therefore the following apply.

$$e = \frac{\dot{\theta}_0{}^{+}}{\dot{\theta}_0{}^{-}} = \sqrt{\frac{\frac{1}{2} I \left(\dot{\theta}_0{}^{+}\right)^2}{\frac{1}{2} I \left(\dot{\theta}_0{}^{-}\right)^2}} = \\ = \sqrt{\frac{mgb\theta_1{}^{+}}{mgb\theta_1{}^{-}}} = \sqrt{\frac{\theta_1{}^{+}}{\theta_1{}^{-}}} = \sqrt{\frac{\theta^{+}}{\theta^{-}}} \tag{6}$$

3 EXPERIMENTAL TESTS

To validate the described theoretical models, an extensive experimental campaign was conducted. The tests were performed by investigating the behaviour of 10cm square base per 30cm height blocks and a 10cm square base per 15cm height two-block configurations made by cellular concrete, a material which allows an easy manufacturing of small pieces, very manageable and that has mechanical properties similar to the travertine, the real material which the blocks of the temples of Paestum are made of.

The experimentation was carried out by considering three different configurations of the initial angle (6°, 12° and 18°) and different materials of the surface where the block relies such as cellular cement (Figure 4), sandpaper sheet and wood.

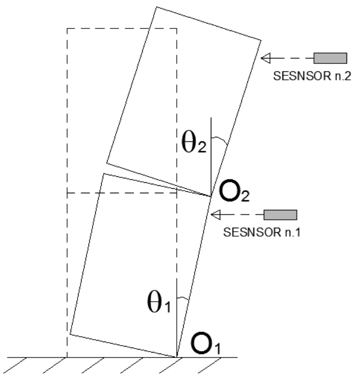

Figure 4: Experimental set-up configuration.

For each considered initial configuration, 10 release tests were conducted, leading to evaluate vibration periods and equivalent viscous ratios by considering the subsequent maximum amplitude evolution of the response.

In the following, the results coming from the configuration with 12° as initial angle of rotation and cellular cement as base interface material are shown. The dynamic motion assessment of the block has been performed by using Micro-Epsilon interferometric laser sensors (Opto ILD 1420-200). An example of the experimental dynamic response is described in Figure 5.

For every single cycle, the principal descriptive parameters of the dynamic behaviour of the single block have been evaluated, i.e. the vibration period, the restitution coefficient and the damping factor.

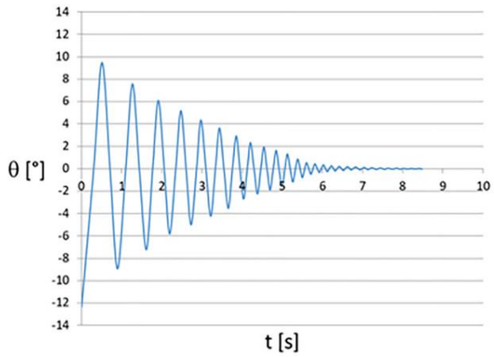

Figure 5: Free response time history example.

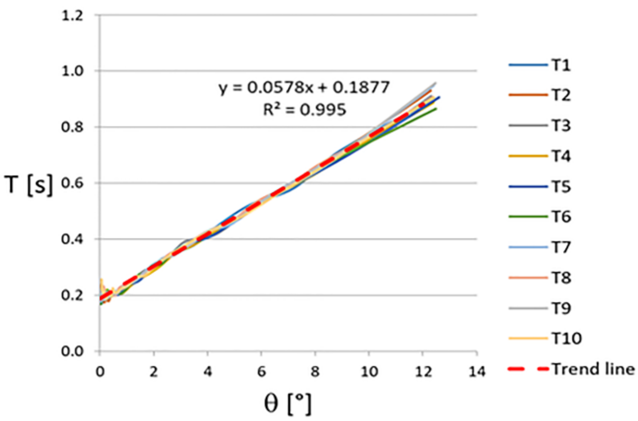

Figure 6.a: Experimental vibration period trends: one-block configuration. (Ti = Test i).

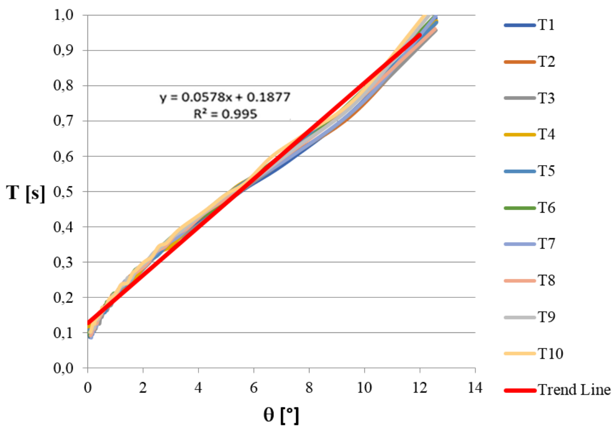

Figure 6.b: Experimental overall vibration period trends: superior block of the two-block configuration. (Ti = Test i).

Figures 6 show the trend of vibration periods obtained for initial configuration $\theta=12°$ in the case of both the considered configurations. In both the cases trend lines, highlighting the correlation factors R, are plotted.

The comparison between the trend lines for the case of one block, obtained by the experimental data, and the theoretical formulation is plotted in the Figure 7.

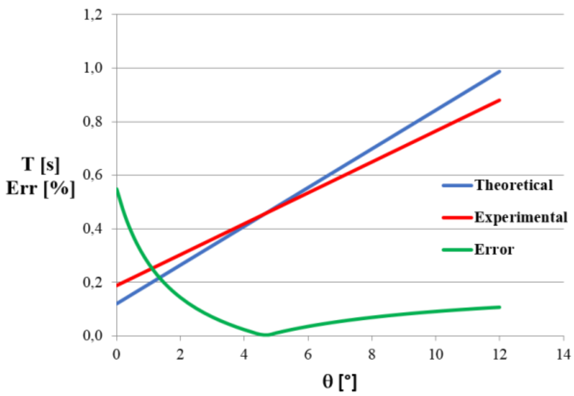

Figure 7: Theoretical and experimental vibration periods comparison – One block configuration. The error is represented by considering 1=100%.

Results highlight an error less than 10% for rotation angles greater than 2°; instead, the error increases exponentially for lower rotation angles. Similar results have been obtained for the two-block configurations. This result can be explained by considering the role of local imperfections on the dynamic behaviour; the lower the response is, the more local imperfections are relevant.

Figures 8 show the estimated restitution coefficients obtained for the configuration with cellular cement as interaction surface, the trend line evaluated for initial angles greater than 2° and the theoretical value (0,85) estimated by the relation (6).

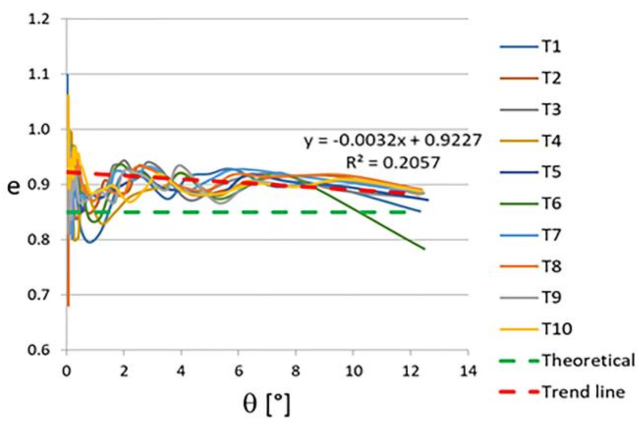

Figure 8.a: One-block configuration: comparison between the experimental and theoretical restitution coefficient.

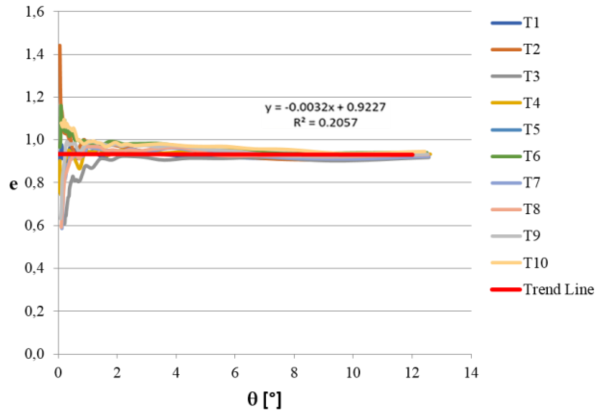

Figure 8.b: Superior block of the two-block configuration: experimental restitution coefficient.

Also in this case similar results are obtained for both the configuration, highlighting errors less than 5% for initial rotation angle greater than 2°.

The experimental results allow assessing the equivalent viscous damping ξ by considering an equivalent linear single degree of freedom (SDOF) oscillator. For the purpose the following relation can be considered (Claugh and Penzien, 2003):

$$\eta = e^{-\frac{2\pi\xi}{\sqrt{1-\xi^2}}} \tag{7}$$

η is the reduction ratio obtained by considering two consecutive maximum response values.

Figures 9.a and 9.b show the experimental equivalent damping factor evaluated for initial angles greater than 2° for both configurations. The trend lines and the theoretical one (5.16%).

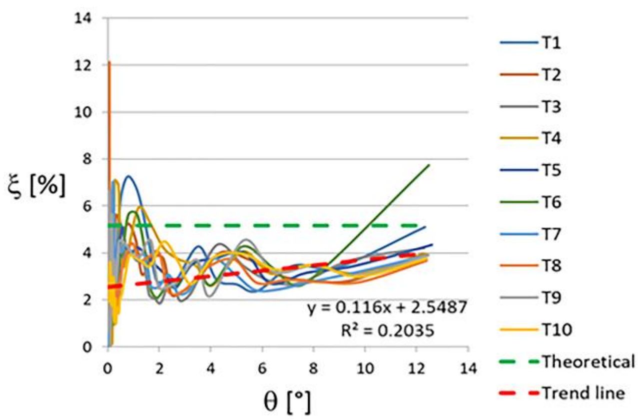

Figure 9.a: One-block configuration: equivalent damping factor trend.

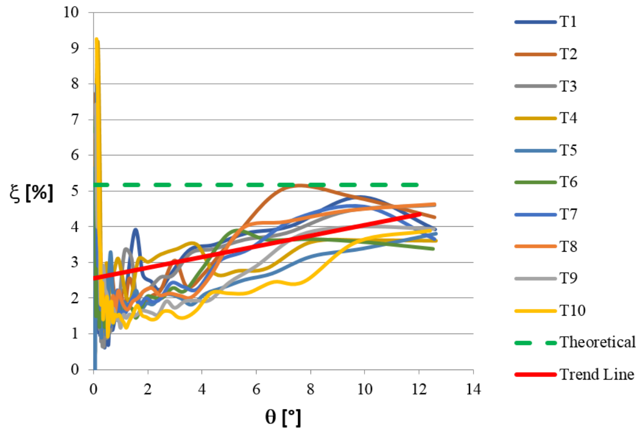

Figure 9.b: Superior block of the two-block configuration: equivalent damping factor trend.

Also for the damping factor, the results obtained for the two configurations are quite similar; in particular, the superior block of the two-block configuration has a trend line slightly more sloped than the one-block configuration.

Results show that the inistial rotation angles do not change the overal assessed behaviour.

4 SEISMIC RISK ASSESSMENT OF COLUMNS OF TEMPLE OF NEPTUNE

To evaluate the seismic safety of structures such as the Greek temple of Neptune, an analysis with a 3D numerical modeling code for advanced geotechnical analysis of soil, rock, ground water, structural support, and masonry was carried out. The software used was *3DEC*, that simulates the response of discontinuous media (such as jointed rock or masonry bricks) that is subject to either static or dynamic loading. The numerical formulation is based on the distinct element method (DEM) for discontinuous modeling (*R. Hart, P. A. Cundall, 1988*).

The set-up of the *3DEC* model was used by considering three basic components of the problem: the geometry (represented through the distinct element model), constitutive links and proprieties of the material with boundary conditions. The 3DEC numerical model, shown in Figure 10, was intended to represent a column of the Neptune Temple constituted of six simply superimposed blocks. The dimensions of the rocks are slightly different between the 36 columns of the peristyle, so a reference column having the average dimensions of the other 36 has been considered.

H =	7,95	[m]
B_{max}=	2,19	[m]
B_{min}=	1,65	[m]
β =	0,034	[rad]
β =	1,95	[°]

Figure 10. *3DEC* Column model of the Neptune Temple.

The contact formulation for the column model is based on multiple contact points, which were located at the vertices and the edge-to-edge intersections. Each contact between adjacent blocks was represented by several sub-contacts, which might be of two types: vertex-to-face and edge-to-edge. The normal stiffness was equivalent to 5.5×10^{10} Pa/m, the tangential stiffness to 5.5×10^{10} Pa/m and the friction angle was assumed to be 37.

Seismic inputs (30 recorded accelerograms) were chosen through the *REXEL v. 3.5* computer-aided code, according to the seismic hazard at the Paestum archaeological site, considering a return period T_R of 2475 years, corresponding to the Limit State of Collapse prevention (SLC) for strategic constructions in the Italian regulations (*NTC – 2018*, New Regulations for Constructions).

The collapse configurations were not imposed, but they were evaluated by considering, in this research phase, scaled seismic records with regard to the PGA parameter.

The figure 11 describes the frequency of safety factors collapse obtained from the 30 considered recorded earthquakes.

To lead a validation of the obtained results, a Non-Linear Static Analysis (NLSA) was considered to evaluate the seismic risk by considering all the possible overturning failure mechanism. In particular, according to the Italian regulations (NTC2018), Capacity Curves of the considered column for different collapse configurations were carried out considering two different safety factor evaluation approaches.

The first was based on the displacements approach, defining the safety factor as the ratio between the half of the base length of the blocks and the top expected maximum displacement evaluated by the NLSA. The second was based on the force approach, defining the safety factor as the ratio between the stable and the overturning moments around the lower vertex.

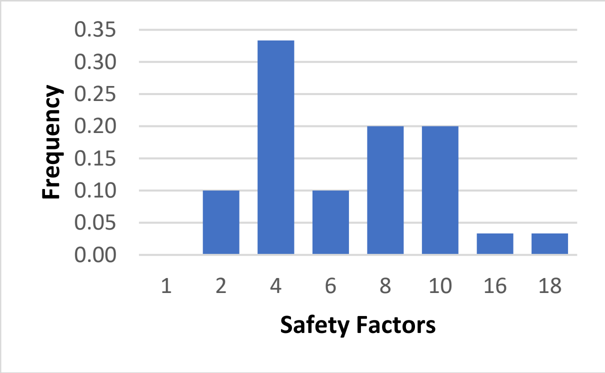

Figure 11: Superior block of the two-block configuration: equivalent damping factor trend.

Table 1 shows the overall obtained results in terms of safety factors (SF) with regard to the 3DEC procedure; regarding the Non-Linear Static Analysis, the minimum safety factors values respectively assessed for the displacement (D) or force (F) approaches are respectively 14,35 and 1,15.

Table 1. 3DEC Safety factors assessment overall results.

	μ_{SF}	σ	$\mu_{SF} + \sigma$	$\mu_{FS} - \sigma$
3DEC outcomes	6,87	3,78	10,65	3,08

As expected the displacement approach lead to greater safety factors than the force approach, while the DEM approach produces results between the upper bounds.

5 CONCLUSIONS

The paper investigated the behaviour of rigid blocks motion by comparing analytical and experimental results for columns made up of rigid blocks.

The obtained experimental results leaded to evaluate restitution ratio parameters, vibrations periods and equivalent damping factors for single and multi-block configurations. Moreover, the experimental results confirm the theoretical relationships for dynamic response described by initial rotation angle greater than 2°, while for smaller angles the response seems to be affected by local imperfections.

The obtained results lead to evaluate by NLSA with displacement approach the safety factor against the overturning collapse mechanism produced by seismic loads as fist attempt.

REFERENCES

UNESCO (2013). *Managing Cultural World Heritage.* ISBN 978-92-3-001223-6.

Applied technology Council (1996). *ATC-40. Seismic Evaluation and Retrofit of Concrete Buildings*. Report No. SSC 96-01 November 1996.

NTC – 2018, *New Regulations for Constructions.*

R. Hart, P. A. Cundall, 1988. *Formulation of a Three-dimensional Distinct Element Model – Part II. Mechanical Calculations for Motion and Interaction of a System Composed of Many Polyhedral Blocks.*

Petti, L. & Marino, I. (2010). *The polar spectrum: a new spatial representation of seismic demand seismic demand.* 14th European Conference on Earthquake Engineering, pp.1-10, Ohrid, 2010. ISBN:9786086518516.

Petti, L., Barone, F., Mammone, A. , Giordano, G., Di Buono A. (2016). *Advanced Methodologies and Techniques for Monuments Preservation: the Trajan Arch in Benevento as a Case of Study*. In 1st International Conference on Natural Hazard & Infrastructure, EACS 2016 – 6th European Conference on Structural Control Sheffield, England: 11-13 July 2016.

Housner, G. W. (1963). *The behavior of inverted pendulum structures during earthquakes.* Bulletin of the Seismological Society of America. Vol. 53, No. 2, pp. 403-417, February 1963.

MINAFO', G., AMATO, G., STELLA, L. (2016). *Rocking behaviour of multi-block columns subjected to pulse-type ground motion acceleration.* The Open Construction and Building Technology Journal, 2016, 10, (Suppl 1: M9) 150-157.

DROMOS, V., ANASTASOPOULOS, I. (2014). *Shaking table testing of multidrum columnd and portals.* Earthquake Engineering & Structural Dynamics, 2014, 43:1703-1723.

YIM, C.S., CHOPRA, A.K. & PENZIEN, J. (1980). *Rocking response of rigid blocks to earthquakes.* Earthquake Engineering & Structural Dynamics, 1980, vol. 8, 565-587.

KONSTANTINIDIS, D. & MAKRIS, N. (2005). *Seismic response analysis of multidrum classical columns.* Earthquake Engineering & Structural Dynamics, 2005, 34: 1243-1270.

Petti L, Sicignano F, Greco D (2017). *Dynamic behaviour assessment of ancient columns through experimental analysis*. In: *XVII Convegno ANIDIS "L'Ingegneria Sismica in Italia"*, 17-21 Settembre, Pistoia, pp. 2534-2541.

Obón Santacana F., Dorka U. E., Nguyen C. K. L, Petti L. (2017). Behaviour of Greek columns during earthquakes with and without tendon system. Lisbon, Portugal

Petti, L., Sicignano F., Greco D. (2018). *Seismic and dynamic assessment of monumental structures made up of rigid blocks: Neptune temple*. In 16th European Conference on Earthquake Engineering, Thessaloniki, 2018.