



# Empirical macro-model for flexure-controlled reinforced concrete columns with plain bars

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## ABSTRACT

In this study, an empirical approach for the nonlinear modelling of flexure-controlled Reinforced Concrete (RC) columns with plain bars is developed. A database of tests on RC columns with plain bars is collected, with varying axial load, material properties, geometry, and longitudinal and transverse reinforcement ratio. The response is analysed in terms of base moment-chord rotation relationship, identifying characteristic points corresponding to yielding, maximum, “ultimate”, and zero resistance conditions. The hysteretic response is analysed, too, evaluating the degradation of unloading and reloading stiffness with imposed inelastic displacement and the “pinching” behaviour. Empirical formulations are developed, analysing the dependence of the investigated parameters on selected potential predictors. The effectiveness of the proposed formulations is evaluated, comparing the modelled response and the predicted deformation capacity with the results obtained applying modelling approaches and capacity models proposed by literature studies and technical codes.

## 1 INTRODUCTION

The development of capacity models for displacement-based assessment and nonlinear modelling of Reinforced Concrete (RC) members is a key issue both for pre-normative research and for seismic vulnerability/fragility analysis of existing buildings. Capacity models are usually developed through an empirical or semi-empirical approach, thereby relying on a collected database of experimental tests. In this study, the deformation capacity of RC columns with plain bars is analysed, starting from a database of experimental tests collected from literature.

Existing models for deformation capacity assessment and modelling of RC columns are usually based on members with deformed bars (e.g. [Elwood et al. 2007](#), [Biskinis and Fardis 2010a-b](#), [Ghannoum 2017](#)). Generally speaking, the reliability and applicability of these models are strictly linked to the database they are based on. The post-elastic response and the corresponding cyclic degradation in members with plain bars can be significantly different compared to members with deformed bars, mainly due to the influence of lower bond capacities ([Verderame et al. 2009a-b](#),

[Melo et al. 2015a](#)) on deformation mechanisms, especially the well-known fixed-end-rotation contribution ([Verderame et al. 2008a-b](#), [Di Ludovico et al. 2014](#), [Melo et al. 2015b](#)). Recognizing these differences, specific empirical-based correction coefficients have been proposed for the deformation capacity assessment of members with plain bars ([CEN, 2005, 2009](#)).

In this study, an empirical macro-model is developed, providing the response envelope of ductile (flexure-controlled) RC columns with plain bars. In addition, the hysteretic response parameters of this type of members are investigated, namely the unloading and reloading stiffness and the “pinching” effect. To this aim, a database of cyclic tests from literature is collected, processed and analysed. Characteristic points of the response envelopes are identified (corresponding to yielding, maximum, “ultimate”, and zero resistance conditions). Potential predictors are investigated and empirical formulations are proposed, based on a statistical analysis of data, for predicting the response envelopes and the hysteretic behaviour.

## 2 STATE-OF-THE-ART

Several approaches have been proposed in literature for the empirical-based deformation capacity assessment and nonlinear modelling of RC elements. This paper aims at contributing to the studies providing an estimate of the expected post-elastic response backbone through empirical formulations based on regression of experimental data. Within this kind of approaches, a first, main distinction has to be made between literature proposals (i) able to account, explicitly, for strength and stiffness degradation due to inelastic cyclic displacements (“degrading” models) and (ii) already including such degradation effects in the predicted response backbone, calibrated on the cyclic experimental response envelope (“non-degrading” models).

Clear advantages of “degrading” models lie in possibility of taking into account the influence of the displacement demand history on the element response, and this is the best modelling option for non-linear dynamic analysis. The larger the inelastic demand, the more “correct” should be the estimate of the response of the element based on “degrading” rather than “non-degrading” models. This is particularly true when approaching element collapse (meant as zero resistance condition). Nevertheless, an accurate and reliable calibration of such a kind of models is not easy at all, as well as their implementation. Probably these are the reasons for the relatively limited availability of this kind of models in literature. On the other side, advantages of “non-degrading” models are the ease of implementation, the possibility of use in non-linear static assessment and – very importantly – their immediate usefulness for pre-normative proposals, once a performance-based assessment criterion is established (e.g., conventional collapse or “ultimate” point assumed at 20% strength drop). Main literature studies adopting the described approaches are briefly recalled as follows.

A reference study adopting the “degrading” approach has been carried out by (Haselton et al. 2008), calibrated on 255 RC columns with deformed bars from the PEER database (Berry et al. 2004). The authors provide empirical formulations defining a trilinear response backbone with yielding, peak and zero resistance points, together with a parameter for modelling the stiffness/strength response degradation; the authors’ proposal is consistent with the cyclic response model proposed by (Ibarra et al. 2005).

Several studies have adopted the “non-degrading” approach; most of these have focused their attention on the deformation capacity

associated to one or more characteristic points evaluated on the backbone of the cyclic experimental response, usually corresponding to conditions as yielding, conventional collapse or “ultimate” (more often), and zero resistance. A reference study was proposed by (Panagiotakos and Fardis 2001): based on a large database of flexure-controlled experimental tests on RC elements, the authors provided formulations for chord rotation capacity at yielding and “ultimate” (at 20% strength drop), the latter through an empirical and a semi-empirical (i.e., based on the plastic hinge length) approach. During the following years, Fardis and co-workers have proposed several adjustments to these formulations, also based on the enlargement of the experimental database, up to (Biskinis and Fardis 2010a-b, Grammatikou et al. 2017).

As far as code provisions for deformation capacity assessment and nonlinear modelling of RC elements are concerned, European standard EC8 substantially adopted Fardis and co-workers’ proposals, including “correction coefficients” for ultimate deformation capacity that have to be used when non-conforming elements are assessed (CEN, 2005, 2009), in order to account for their lower ductility compared to the seismically detailed elements of the database used for the empirical calibration of the original formulations (Panagiotakos et al. 2002, Biskinis and Fardis 2010b). Regarding US standards, ASCE/SEI 41-13 (ASCE 2013) provides a procedure for the classification of the expected failure mode (Flexure, Flexure-Shear, Shear, Inadequate development or splicing) and, accordingly, empirical deformation capacity parameters depending on different element characteristics and calibrated to satisfy a target failure probability established for each failure mode. Such provisions were first incorporated by ASCE/SEI 41-06 – Supplement 1 (ASCE 2007b), an update of ASCE/SEI 41-06 (ASCE 2007a), based on a proposal by (Elwood et al. 2007). Recently, ASCE/SEI 41 provisions on modelling and capacity parameters for RC columns were re-evaluated in (Ghannoum and Matamoros 2014, Ghannoum 2017); the authors proposed continuous (not failure mode-dependent) empirical expressions providing a median estimate of modelling parameters, and also updated acceptance criteria. The New Zealand Society for Earthquake Engineering (NZSEE) Guidelines for The Seismic Assessment of Existing Buildings (NZSEE 2017) adopt a procedure for the estimation of the deformation capacity at Ultimate Limit State (ULS) based on plastic hinge length approach, referring to (Priestley et al. 2007). The

expected reduction in post-elastic deformation capacity due to construction details typical of older practice is taken into account by assuming a reduced value of the plastic hinge length. Moreover, the expected drift at the initiation of longitudinal bar buckling, which may limit the calculated deformation capacity at ULS for flexure-controlled columns, is evaluated through the empirical formulation proposed by (Berry and Eberhard 2005).

With regard to RC elements with plain bars, despite the widespread presence of this reinforcement typology in RC structures built in the Mediterranean area up to the 1970s and in North American countries and New Zealand until 1950, relatively few analytical studies concerning the assessment of their deformation capacity are present in literature. As far as code provisions are concerned, specific expressions for the above-mentioned correction coefficients are provided by EC8 (CEN, 2005, 2009) for elements with plain bars. Such coefficients, mainly accounting for the reduction of the ultimate chord rotation capacity due to insufficient lap splice length, have been re-evaluated by different authors (Biskinis and Fardis 2010b, Verderame et al. 2010, Melo et al. 2015b). Moreover, the effectiveness of current ASCE/SEI 41 provisions for elements with plain bars has been evaluated in (Ricci et al. 2013), highlighting a significant conservatism. Finally, as mentioned above, the NZSEE Guidelines account for the influence of plain longitudinal reinforcement on post-elastic deformation capacity reducing the assumed plastic hinge length, in order to reflect the observation that the presence of construction details typical of older practice (including low longitudinal reinforcement ratio and inadequately constructed cold joints, too) can lead to a single crack opening and a concentration of tensile strain demand in the reinforcement, thus limiting the development of the plastic hinge length.

Recently, specific proposals for modelling and assessment of deformation capacity of elements with plain bars were developed by (O'Reilly and Sullivan 2017) and (Grammatikou et al. 2018). (O'Reilly and Sullivan 2017) proposed a nonlinear modelling approach for the seismic assessment of the existing Italian RC frame structures; within this approach, the flexure-controlled behaviour of beam/column elements with plain bars was modelled assuming a lumped plasticity approach, with the plastic hinge length proposed by (Paulay and Priestley 1992) and a trilinear moment-curvature response backbone with yielding, peak and zero resistance points. This approach was based (and validated) on a database of experimental tests on flexure-controlled RC

columns with plain bars. Cyclic degradation effects were not modelled, thus relying only on the post-peak negative stiffness to simulate the loss of strength. (Grammatikou et al. 2018), based on a database of tests on RC elements with plain bars, developed different expressions, both mechanical-based and empirical, to evaluate the deformation capacity at yielding and ultimate, accounting for the possible presence of FRP strengthening, too. At yielding, the authors proposed a mechanical approach to evaluate the chord rotation based on an idealized strut-and-tie mechanism, assuming a linear variation of the stress in the longitudinal reinforcement between the yield stress at the section where the moment attains its yield value and the ends of the bar. Chord rotation at ultimate – i.e., 20% strength degradation – was calculated as the sum of the chord rotation at yielding and of a plastic part; for the evaluation of the latter, both a mechanical-based and an empirical approaches were proposed.

### 3 EXPERIMENTAL DATABASE

In this study, the deformation capacity and the hysteretic response of RC columns with plain bars was analysed based on the observation of experimental data provided by different authors in literature. In the following, the collection of the database and process of extraction of the data of interest are described. The aim of this extraction is to analyse the main characteristic points of the base moment-chord rotation response envelopes of the tests.

#### 3.1 *Collection and processing of experimental data*

Experimental tests on RC columns with plain bars were collected. Reported force-displacement relationships were digitized. Only cyclic tests were considered, in order to (although approximately) account for the effect of cyclic degradation on the inelastic response.

Tests characterized by a failure in shear (prior to or after flexural yielding) were excluded, thus considering only ductile (flexure-controlled) elements. Tests with unsymmetrical reinforcement, representative of beams, were not considered. Both tests with continuous and with lap-spliced longitudinal reinforcement were considered; among these, in particular, tests with 180-degree (representative of older construction practices) end-hooked lap-splicing were considered; tests with lapped straight bars, which can experience premature bond failures (particularly due to the poor bond performance of

plain reinforcement) instead of flexural failures, were not considered. Based on the analysis of the experimental setup and axial load application scheme reported by the authors, P-Delta effects were removed, i.e., if necessary the response was corrected in order to be consistent with “Case I” reported in (Berry et al., 2004); therefore, the softening behaviour in the base moment-chord rotation relationships analysed herein is due only to mechanical degradation phenomena, and not to geometric non-linearity effects due to the presence of axial load.

The collected tests are characterized by  $0 \leq \nu \leq 0.63$ ,  $10.6 \leq f_c \leq 30.3$  MPa,  $275 \leq f_y \leq 405$  MPa,  $331 \leq f_{yw} \leq 430$  MPa,  $0.008 \leq \rho_l \leq 0.029$ ,  $0.0010 \leq \rho_w \leq 0.0039$ ,  $2.7 \leq L_s/d \leq 7.6$ .

The database collected in this study, which is described more in detail in (Verderame and Ricci 2018), is wider than the database used by (O’Reilly and Sullivan 2017), which is made of 23 tests on RC columns with plain bars (in the major part tested at the University of Naples Federico II), both cyclic and monotonic, with continuous or end-hooked lap-spliced longitudinal reinforcement. On the contrary, the database collected in this study is significantly smaller compared to the database used by (Grammatikou et al. 2018), which is made of almost 200 tests on RC elements with plain bars, including elements reinforced with Fiber Reinforced Polymer (FRP) jackets. Among the tests on unretrofitted specimens, the ones satisfying the abovementioned criteria were included in this study, too.

### 3.2 Data extraction

A base moment-chord rotation multilinear relationship was adopted for describing the envelope of the inelastic cyclic response. To this aim, first of all, characteristics points/conditions were defined, namely:

- yielding, corresponding to the attainment of the theoretical moment at first yielding,  $M_y$ , calculated with a fibre analysis; if the maximum measured moment  $M_{max}$  was not at least 7% larger than  $M_y$ , the yielding point was identified with a moment equal to 0.80 times the peak resistance,  $M_{max}$ , according to (Elwood and Eberhard 2009);
- peak resistance;

- “ultimate” condition, corresponding to a 20% strength drop on the envelope of the response curve;
- zero-resistance, evaluated extrapolating to zero the line interpolating the extreme points of imposed displacement cycles of the softening branch of the response envelope.

The following parameters were adopted to identify the abovementioned characteristic points of the response envelope. Note that, different from (Verderame and Ricci 2018) but as already shown in (De Risi et al. 2019), predictive equations were developed for “partial” deformability contributions, as shown in Figure 1:

- ratio between section effective stiffness (secant-to-yielding)  $EI_{eff}$  and section the gross section stiffness  $EI_g$ ,  $EI_{eff}/EI_g$ , and theoretical moment at first yielding,  $M_y$ , calculated as described above;
- peak resistance,  $M_{max}$ , and corresponding post-yielding (plastic) chord rotation,  $\theta_{max}^{pl}$ ;
- post-capping “ultimate” chord rotation,  $\theta_{ult}^{pc}$ ;
- post-ultimate chord rotation at zero-resistance,  $\theta_0^{pu}$ , and corresponding softening stiffness toward zero resistance,  $K_0$ .

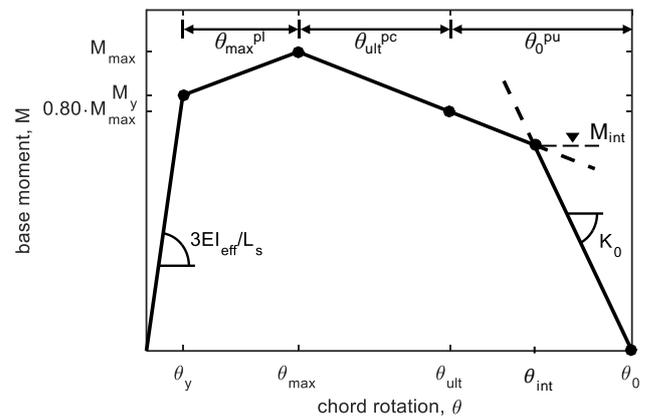


Figure 1. Characteristic points and assumed parameters of the base moment-chord rotation response envelope.

## 4 METHODOLOGY

The difficulties in predicting the nonlinear response of RC members (and thereby their deformation capacity), accounting for different deformation mechanisms, with enough confidence, especially in large inelastic field, makes the empirical approach attractive, together with

further advantages as reliability, robustness, ease of implementation and the direct validation based on experimental test results.

Different regression methodologies can be adopted to derive empirical formulations starting from observed data from experimental tests. Generally speaking, the first step consists of the selection of potential predictive parameters based on mechanical-based engineering judgment and previous studies; then, trends of the output (predicted) variable with the potential input (predictor) variables are observed; finally, a functional form is assumed and regression coefficients are derived.

In the following, the methodology adopted for deriving the empirical formulations proposed in this study is briefly illustrated.

#### 4.1 Potential predictive parameters

A set of potential predictive parameters was selected, based on previous literature studies and their expected mechanical influence; they are reported as follows.

- Concrete compressive strength,  $f_c$ ;
- Longitudinal steel yield strength,  $f_y$ ;
- Axial load ratio,  $v$ ;
- Shear span-to-depth ratio,  $L_s/d$ ;
- Transverse reinforcement spacing-to-depth ratio,  $s/d$ ;
- Transverse reinforcement spacing-to-longitudinal bar diameter ratio,  $s/d_b$ ;
- Rebar buckling coefficient,  $s_n=(s/d_b) \cdot (f_y/100)^{0.5}$ ;
- Geometrical,  $\rho_l$ , and mechanical,  $\omega_l$ , longitudinal reinforcement ratio;
- Geometrical,  $\rho_w$ , and mechanical,  $\omega_w$ , transverse reinforcement ratio;
- Compression-to-tension longitudinal reinforcement ratio,  $\omega'/\omega$ ;
- Splice length-to-longitudinal bar diameter ratio,  $l_o/d_b$ ;
- Fixed-end-rotation coefficient,  $(l_{ba} \cdot d_b)/(d \cdot f_c^{0.5})$ , with  $l_{ba}$  equal to the longitudinal reinforcement anchorage length.

#### 4.2 Statistical data analysis

Predictive equations were derived carrying out linear least squares regressions between the natural logarithm of the output variable and the input variables (assumed in their natural or logarithmic form, or not present). The final (reduced) form of

each predictive equation was selected among the ones with the minimum number of input variables but deemed as “statistically equivalent” – based on F-tests – to the corresponding “complete” model, including all of the input variables. If possible, predictor variables were transformed in linear form. Predictive equations for the parameters listed above are reported as follows, along with mean, median and Coefficient of Variation (CoV) of the observed-to-predicted ratio. Note that, in some cases, predictive equations are re-derived in an alternative form excluding the “fixed-end-rotation coefficient”, that in some cases (e.g., continuous longitudinal reinforcement passing through beam-column joints) could not be easily determined.

## 5 EVALUATION OF THE RESPONSE ENVELOPE

In the following, the results of the statistical data analysis carried out according to the methodology described in Section 4.2 are reported. For each response parameters extracted from experimental data (Section 3.2), the observed correlation with the assumed potential predictive parameters (Section 4.1) is reported.

The ratio between effective and gross section stiffness,  $EI_{eff}/EI_g$  can be calculated as reported in Equation 1 (obs.-to-pred. ratio: mean=1.04, median=1.04, CoV=0.25):

$$EI_{eff}/EI_g = 0.074 \cdot 8.1^v \cdot (1 + 0.30 \cdot L_s/d) \quad (1)$$

The post-yielding (plastic) chord rotation,  $\theta_{max}^{pl}$  can be calculated as reported in Equation 2 (obs.-to-pred. ratio: mean=1.07, median=1.05, CoV=0.35):

$$\theta_{max}^{pl} = 0.0026 \cdot 0.106^v \cdot (1 + 1.20 \cdot l_{ba} d_b / d \sqrt{f_c}) \cdot (0.49 + 0.51 \cdot \min(l_o/d_b, 50) / 50) \quad (2)$$

The post-yielding (plastic) chord rotation,  $\theta_{max}^{pl}$ , excluding the “fixed-end-rotation coefficient” can be calculated as reported in Equation 3 (obs.-to-pred. ratio: mean=1.06, median=1.05, CoV=0.36):

$$\theta_{max}^{pl} = 0.0062 \cdot 0.17^v \cdot (1 + 0.28 \cdot L_s/d) \cdot (0.49 + 0.51 \cdot \min(l_o/d_b, 50) / 50) \quad (3)$$

For the calculation of the peak resistance,  $M_{max}$ , a simple mean value could be proposed attempting to predict the  $M_{max}/M_y$  ratio, i.e.,  $M_{max}/M_y = 1.14$  (obs.-to-pred. ratio: CoV=0.14), with  $M_y$  calculated by means of a section fibre analysis.

The post-capping “ultimate” chord rotation,  $\theta_{ult}^{pc}$  can be calculated as reported in Equation 4 (obs.-to-pred. ratio: mean=1.09, median=1.05, CoV=0.39):

$$\theta_{ult}^{pc} = 0.033 \cdot 0.0013^v \cdot \omega_{sw}^{0.51} \cdot (l_{ba}d_b/d\sqrt{f_c})^{1.61} \quad (4)$$

The post-capping “ultimate” chord rotation,  $\theta_{ult}^{pc}$ , excluding the “fixed-end-rotation coefficient” can be calculated as reported in Equation 5 (obs.-to-pred. ratio: mean=1.12, median=1.00, CoV=0.49):

$$\theta_{ult}^{pc} = 0.0082 \cdot 0.0034^v \cdot \omega_{sw}^{0.63} \cdot (1 + 10.4 \cdot L_s/d) \quad (5)$$

The post-ultimate chord rotation at zero resistance,  $\theta_0^{pu}$  can be calculated as reported in Equation 6 (obs.-to-pred. ratio: mean=1.12, median=0.98, CoV=0.61):

$$\theta_0^{pu} = \min(0.0022 \cdot 0.0050^v \cdot 227^{(\rho_w \cdot 100)} \cdot (1 + 3.74 \cdot l_{ba}d_b/d\sqrt{f_c}); 0.11) \quad (6)$$

The post-ultimate chord rotation at zero resistance,  $\theta_0^{pu}$ , excluding the “fixed-end-rotation coefficient” can be calculated as reported in Equation 7 (obs.-to-pred. ratio: mean=1.14, median=0.97, CoV=0.70):

$$\theta_0^{pu} = \min(0.031 \cdot 0.015^v \cdot 243^{(\rho_w \cdot 100)}; 0.11) \quad (7)$$

To avoid modelling issues, it should be assumed in addition that, as reported in Equation 8:

$$\theta_0^{pu} \leq 4\theta_{ult}^{pc} \quad (8)$$

The softening stiffness toward zero resistance,  $K_0$  can be calculated as reported in Equation 9 (obs.-to-pred. ratio: mean=1.31, median=1.01, CoV=0.63):

$$K_0 = \max(29 \cdot 407^v \cdot (\rho_w \cdot 100)^{-1.65}; 700) \quad (9)$$

To avoid modelling issues, it should be assumed in addition that, as reported in Equation 10:

$$K_0 \geq \frac{0.80M_{max}}{\theta_0 - \theta_{ult}} \quad (10)$$

The intersection point reported in Figure 1 associated with base moment  $M_{int}$  and chord rotation  $\theta_{int}$  can be defined by means of Equations 11 and 12.

$$\theta_{int} = \frac{K_0\theta_0 - \frac{0.20M_{max}}{\theta_{ult} - \theta_{max}}\theta_{max} - M_{max}}{K_0 - \frac{0.20M_{max}}{\theta_{ult} - \theta_{max}}} \quad (11)$$

$$M_{int} = \left(1 - 0.20 \frac{\theta_{int} - \theta_{max}}{\theta_{ult} - \theta_{max}}\right) M_{max} \quad (12)$$

## 6 EVALUATION OF THE HYSTERETIC RESPONSE

In this section, the hysteretic response of RC columns with plain bars is investigated based on the experimental database presented in section 3.

In general, a structural member under cyclic (such as seismic) action exhibits an imposed strain-consequent stress response characterized by unloading and reloading phases. For inelastic elements, the unloading and reloading phases are governed by complex rules mainly depending on the maximum deformation demand, on the experienced number of unloading-reloading cycles and on the dissipated energy.

Considering the complexity of these phenomena, but also the key role of the hysteretic response of members in the nonlinear dynamic assessment of structures, different modelling strategies and simplified hysteretic rules have been defined in the past and implemented in structural computation codes to control the hysteretic

response of structural members. Such hysteretic rules are completely defined by means of parameters that the user should set to obtain a specific hysteretic response, potentially similar to that experimentally observed.

For example, the unloading stiffness of structural members is expected to be equal to the initial elastic stiffness only in the first loading stages. Then, at increasing (inelastic) deformation demand, the unloading stiffness is expected to reduce. So, the reduction of unloading stiffness should be reproduced by an appropriate hysteretic rule (or, in other words, by a sort of “model function”) relating it to the deformation demand/dissipated energy. However, the *rate* at which the unloading stiffness reduces at increasing deformation demand/dissipated energy is defined by means of parameters introduced in the general hysteretic rule/model function. As above stated, the user should set such parameters based on experimental evidences.

In this study, it is suggested to model the lateral response of RC columns with plain bars by using the Pinching4 Material, as also done for RC members in other studies proposed in the literature (e.g., [LeBorgne and Ghannoum 2014](#)). The Pinching4 Material implemented in OpenSees ([McKenna et al. 2004](#)) is a generalized stress-strain relation characterized by a 4-point response envelope and by hysteretic rules allowing the reproduction of the variation at increasing deformation demand/dissipated energy of:

1. the unloading stiffness;
2. the reloading stiffness;
3. the maximum stress demand corresponding to a certain strain demand;
4. the so-called “pinching” effect;

For example, for what concerns the degradation of the unloading stiffness,  $k$ , the degradation function reported in Equation 13 applies:

$$k_i = k_0(1 - \delta k_i) \quad (13)$$

In Equation 13,  $k_i$  is the unloading stiffness at the  $i$ -th step. It is equal to  $k_0$ , the initial unloading stiffness in case of no damage, times  $1 - \delta k_i$ , with  $\delta k_i$  calculated as reported in Equation 14

$$\begin{aligned} \delta k_i &= gK_1(d_{max})^{gK_3} \\ &+ gK_2 \left( \frac{E_i}{E_{monotonic}} \right)^{gK_4} \leq gKLim \end{aligned} \quad (14)$$

In Equation 14,  $gK_i$  ( $i=1, \dots, 4$ ) and  $gKLim$  are the unloading stiffness hysteretic rule parameters. Through these parameters,  $\delta k_i$  depends on the deformation history (represented by  $d_{max}$ ) and by the energy accumulation (represented by  $E_i/E_{monotonic}$ ). Further details on the general rules adopted to reproduce the hysteretic variation of the above-listed response parameters can be found in ([Lowe et al. 2004](#)). However, it should be noted that:

1. the unloading stiffness degradation is defined by five parameters ( $gK_i$ , with  $i$  ranging from 1 to 4 and  $gKLim$ ), as shown above;
2. the reloading stiffness degradation is defined by five parameters ( $gD_i$ , with  $i$  ranging from 1 to 4 and  $gDLim$ ), too;
3. the maximum stress degradation is defined by five parameters ( $gF_i$ , with  $i$  ranging from 1 to 4 and  $gFLim$ ), too;

The reloading stiffness and maximum stress degradation are calculated by means of formulations analogous to Equation 13 and 14. In addition,

4. the pinching effect is reproduced by means of three parameters ( $rDisp$ ,  $rForce$  and  $uForce$ ) potentially different for the positive and for the negative loading direction.

In this section, the values of  $gK_i$ ,  $gD_i$ ,  $rForce$ ,  $rDisp$  and  $uForce$  are calibrated based on the experimental response of RC columns with plain bars subjected to cyclic tests. The assessment of the above parameters for each test has been performed by assuming  $gKLim$  and  $gDLim$  equal to 1 and the “damage type” parameter to “energy”, i.e., by accounting for both the effect of the displacement history and the effect of the energy accumulation. The evaluation of  $gF_i$  is not performed (i.e., such factors should be assumed equal to 0), because the proposed response envelope, which has been calibrated in section 5 on the response envelope of members subjected to cyclic loading, already includes the strength

degradation effects in the predicted response backbone.

The reader should be aware that the values of the above-mentioned parameters are significantly different from test to test. Different attempts showed the extreme complexity of defining empirical formulations relating such parameters with the geometrical and mechanical characteristics of the specimens whose response is collected in the database. For this reason, the proposal of robust and reliable formulations of this type is left to future studies. The mean, the median and the CoV of the values of the above-mentioned hysteretic parameters determined for each experimental tests are reported in Table 1. A comparison of the experimental and predicted (with median values of the hysteretic parameters) cyclic response of some test specimens is reported in Figure 2.

Note that in Figure 2 (as well as in Figure 3), in order to evaluate the model effectiveness avoiding the bias due to the difficulties in prediction of strength (particularly because the prediction of strength at first yielding ( $M_y$ ) is made on a theoretical basis, and the prediction of maximum strength depends on the estimated  $M_y$ ),

comparisons are carried out assuming as maximum strength the observed  $M_{max}$ .

Table 1. Mean, median and CoV of the hysteretic response parameters.

Parameter	Mean	Median	CoV
gK1	0.26	0.004	1.90
gK2	0.10	0.05	1.24
gK3	0.79	0.81	0.18
gK4	0.62	0.69	0.57
gD1	0.08	0.06	1.61
gD2	0.17	0.06	2.01
gD3	0.40	0.57	0.89
gD4	0.47	0.68	0.77
rDisp <sup>(*)(**)</sup>	0.80	-	-
rForce <sup>(**)</sup>	0.87	0.87	0.05
uForce <sup>(**)</sup>	-0.14	-0.20	2.56

(\*): Fixed value

(\*\*): Equal for positive and negative loading direction

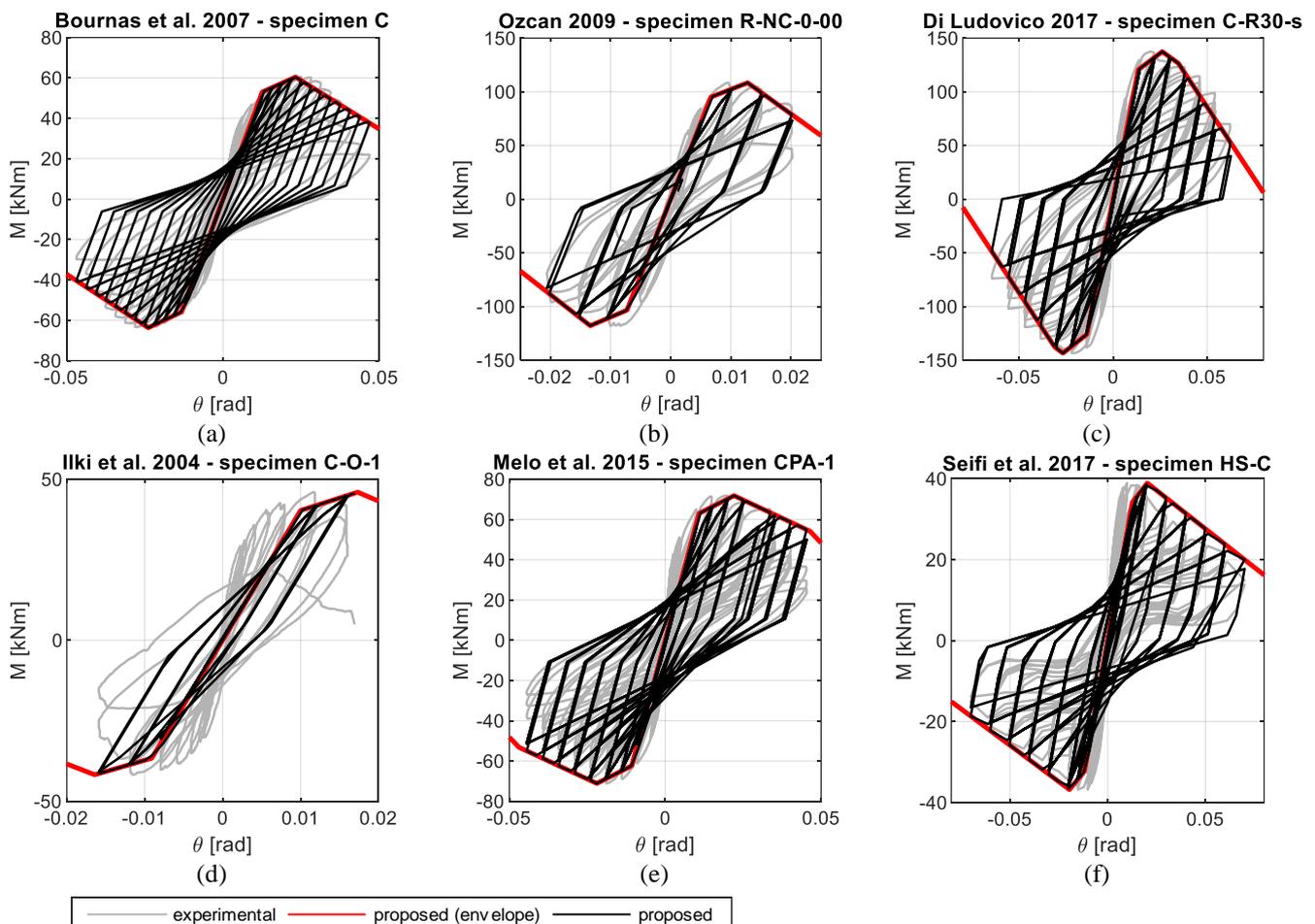


Figure 2. Three best (a-b-c) and three worst (d-e-f) predictions of the proposed hysteretic model.

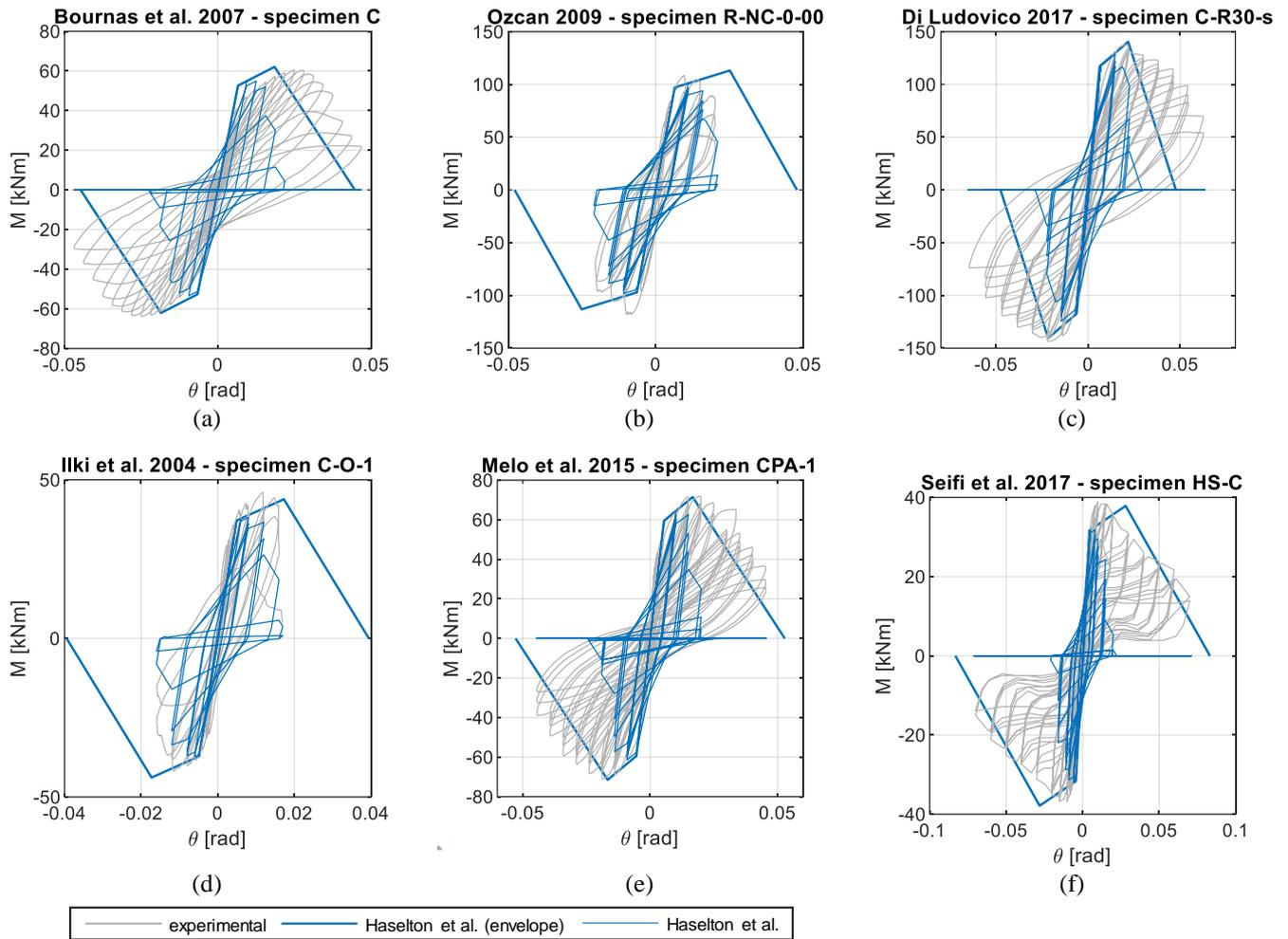


Figure 3. Experimental response of the specimens selected for Figure 2 compared with the prediction by the model by (Haselton et al. 2008).

In Figure 3, the experimental response of the specimens selected to check the effectiveness of the proposed hysteretic model is compared with the prediction of the empirical-based macro-model by (Haselton et al. 2008)’s model, which is dedicated to the prediction of the lateral response of RC columns with deformed bars. Generally speaking, the model by (Haselton et al. 2008) overestimates the cyclic degradation observed in the database tests. From a phenomenological point of view, this could be reasonably expected, since the lower bond capacity of plain longitudinal bars leads to more concentrated crack pattern – up to a single crack opening at element’s end – consistent with the well-known significant influence of fixed-end-rotation mechanism on post-elastic deformation capacity – often referred to as a “rocking” mechanism; on the contrary, in elements with deformed bars a more spread crack pattern is observed, which can lead, especially under cyclic inelastic imposed displacements, to a more significant and widespread damage to concrete and,

thereby, to a more severe degradation in the global lateral load-displacement response, both with increasing imposed displacement and dissipated energy.

## 7 CONCLUSIONS

An empirical macro-model for the prediction of inelastic response of flexure-controlled RC columns with plain bars was proposed. To this aim, a database of cyclic tests was collected, parameters identifying the characteristic point of the response envelope were identified, and a regression analysis was performed in order to derive empirical formulations predicting these parameters. The proposed equations allow modelling the inelastic response up to complete collapse (zero-resistance condition). The proposed macro-model includes hysteresis rules calibrated on the experimental data accounting for unloading and reloading stiffness degradation, as well as for “pinching” effect.

The proposed macro-model can be used both for deformation capacity assessment and for nonlinear modelling, thus representing a useful tool for seismic analysis of existing RC frames with plain reinforcing bars, properly accounting for the specific response characteristics of this kind of members.

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