A simplified formulation to assess shear capacity of circular RC cross-sections

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ABSTRACT
Reinforced concrete (RC) members with circular cross-sections are widely used in structural and geotechnical engineering (e.g. columns in frame structures, foundation piles, etc.). Generally, for such members, the analysis is more complex than for rectangular cross-sections and the problem is not sufficiently investigated in literature. Circular shapes and uniform distribution of reinforcement along the perimeter cause some difficulties for a simple assessment of bending and shear capacity. This paper discusses a new formulation to estimate the shear strength of RC elements with circular cross sections investigating the cases with and without shear reinforcements. The proposed formulations are based on the significant contribution (improvement) given by the longitudinal bars homogenously distributed along the cross-section circumference (i.e. dowel effect) to shear strength.

1 INTRODUCTION

European and Italian codes do not give specific information regarding the shear strength of reinforced concrete (RC) elements with a circular cross-section. However, circular columns are commonly used in buildings but especially in bridge piers, and the estimation of the shear strength is of paramount importance for capacity seismic design. According to the Italian code (NTC 2018), for members with vertical shear reinforcement, the shear resistance V_{Rd} of generic cross-section RC elements is assumed equal to the minimum between concrete resistance in compression and shear reinforcement resistance, estimated according to the truss model with variable compressive strut inclination angles. This approach is easily applicable for rectangular and square cross-sections, but cause some difficulties, for a circular shape with uniform distribution of reinforcement along the perimeter.

Literature commonly suggests the use of the Mörsch truss analogy model assumption or the truss model with variable compressive strut inclination angle to estimate the shear resistance of RC circular cross-section with stirrups.

Specifically, in Ang et al 1989 the shear resistance for RC circular cross-section was estimated assuming 45 degrees crack in concrete. Subsequently, authors used the mean value theorem for integrals, to estimate the tension for each steel vertical stirrup.

In 1993 Clarke and Birjandi suggested to use for circular cross-sections the same formulation given by British Codes of Practice for rectangular cross-sections.

Priestley et al. proposed a modification of the Ang et al method. They studied the arch mechanism with axial action to estimate the shear resistance.

Subsequently, Kowalsky and Priestley, (2000) further updated the Ang et al method assuming the effect of aspect ratio and longitudinal steel ratio on the strength of the concrete shear resisting mechanism.

In 2001, Dancygier proved that Ang et all method was satisfactory only for circular cross section diameters which were four times the stirrups spacing. In all other cases, the Ang et al method gives inaccurate values. The relative percentage error can exceed 50 %.

Kim and Mander (2005), in agreement with Dancygier, proposed a reduction factor of the shear resistance when the number of stirrups in a single crack is greater than 5.

Merta et al, (2003, 2004 and 2006), proposed an analytic approach to estimate the shear resistance in circular RC cross sections. They were the first authors to take into account increased resistance as a result of stirrup curvature.

Fiore et al (2014) estimated the shear resistance from a statistic investigation of experimental results given by literature. They proposed five expressions including many parametrical coefficients.

In Thamrin et al (2017) experimental results of circular elements are compared with formulations for rectangular section of ACI-318M-14.

This paper aims to give specific formulations for circular cross sections and in particular a simplified method for shear capacity assessment, as an alternative to the truss mechanism or in general the expressions including many parametrical coefficients.

2 THE SHEAR STRENGTH IN RC CIRCULAR CROSS-SECTIONS

As known, the shear resistance of RC elements is improved by mechanisms as an aggregate interlock, cantilever action, dowel action-effect, arching action, etc. These mechanisms give a shear strength in elements without shear reinforcement.

Fig.1 shows that the dowel action given by longitudinal bars is greater for circular cross-sections than rectangular ones. In fact, the uniform distribution of the bars along the cross-section perimeter gives a more effective action against the concrete cover expulsion.

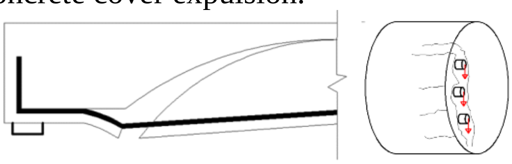

Figure 1. Overview of dowel action-effect contribute of sub-vertical longitudinal bars

In addition, the longitudinal bars reduce the crack width. In fact, they sew cracks along the cross-section perimeter. Fig. 2 shows an overview of this mechanism.

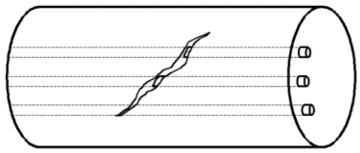

Figure 2. Sub-vertical longitudinal bar action on the crack opening.

For these reasons, the increase in shear strength due to the dowel action of sub vertical longitudinal bars is more effective than in case of rectangular cross sections. In addition, generally, the quantity of longitudinal bars around the circular cross-section circumference is greater than along the rectangular cross-section perimeter, increasing the shear strength of circular cross-sections.

3 THE PROPOSED METHOD

In recent years, researchers have carried out experiments with the aim to propose easy and simplified formulations to estimate the RC circular cross-section shear resistance. In particular, Capon e de Cossio (1965), Clarke e Birjandi (1993), Kim (2000), Collins et al. (2002) presented a database composed by results obtained from 85 different experiments on RC circular cross-sections.

Table 1 lists experimental results of 35 cases regarding beams with a circular cross-section without shear reinforcements. Table 2 gives results of experimental tests on 50 beams with shear stirrups. Results are given in terms of shear strength V_R^{test}.

Tables 1 and 2 also give:

- the concrete circular cross-section diameter (D) that ranges from 150 mm to 500 mm;
- the geometrical percentage of longitudinal bars (ρ_l) that ranges from 0.9% to 5.6%;
- the geometrical percentage of shear stirrups (ρ_w) that range from 0.1% to 0.45%;
- the shear stirrups spacing (s) that ranges from 75 mm to 250 mm;
- the cylinder compressive concrete stress (f'_c) that ranges from 13 to 50 MPa;
- the yield stress of longitudinal steel bars (f_{yl}) that ranges from 400 MPa to 500 MPa;
- the yield stress of steel stirrups (f_{yw}) that range from 250 MPa to 445 MPa;

This database was used to develop the formulations proposed in this paper.

Table 1. Geometry and mechanic parameters of RC elements without shear reinforcements

References	#	*D* mm	f'_c Mpa	f_{yl} Mpa	ρ_l %	V_R^{test} kN
Clarke & BirJandi (1993)	1	300	22.7	500	0.89	65
	2	300	22.8	500	2.28	91
	3	300	22.8	500	2.28	97
	4	300	44	500	2.28	129
	5	300	44	500	2.28	109
	6	300	26.7	500	5.56	148
	7	300	26.7	500	5.56	130
	8	300	43.6	500	5.56	152
	9	300	43.6	500	5.56	148
	10	300	31.2	500	3.56	86
	11	300	29.7	500	3.56	90
	12	300	20.9	500	3.56	98
	13	300	21.6	500	5.56	116
	14	300	34.8	500	5.56	125
	15	300	37.7	500	5.56	125
	16	300	34.9	500	5.56	136
	17	500	34	500	2.56	236
	18	500	33.5	500	2.56	234
	19	500	33.5	500	2.56	222
	20	500	29.4	500	3.84	234
	21	500	30.6	500	3.84	281
Capon &de Cossio (1965)	22	247	25.6	400	2.12	46.5
	23	246	29.2	400	2.14	49
	24	252	46.1	400	3.06	71.6
	25	251	44.4	400	3.08	67.7
	26	251	29.6	400	3.08	70
	27	252	30.6	400	3.06	77
	28	251	13.4	400	3.08	47.5
	29	252	23.7	400	3.06	45.8
	30	251	24.8	400	3.08	47
	31	252	24.9	400	3.06	56.8
	32	251	28.7	400	3.08	53
	33	251	13.7	400	3.08	59
	34	252	20.7	400	1.18	50.5
Kim (2000)	35	445	30.8	460	3.86	212

Table 2. Geometry and mechanic parameters of RC elements with shear reinforcements

References	#	D [mm]	f'_c [MPa]	f_{yl} [MPa]	ρ_l [%]	f_{yw} [MPa]	ρ_w [%]	s [mm]	V_R^{test} [kN]
Clarke & BirJandi (1993)	1	152	28	500	2.2	300	0.37	100	45
	2	152	28	500	2.2	300	0.37	100	46
	3	152	28	500	2.2	300	0.37	100	38
	4	300	24.1	500	5.6	300	0.22	150	186
	5	300	24.1	500	5.6	300	0.22	150	188
	6	300	23.8	500	5.6	300	0.45	75	211
	7	300	23.8	500	5.6	300	0.45	75	239
	8	300	48.4	500	5.6	300	0.22	150	227
	9	300	48.4	500	5.6	300	0.22	150	228
	10	300	50.5	500	5.6	300	0.45	75	279
	11	300	50.5	500	5.6	300	0.45	75	288
	12	300	24.3	500	3.6	300	0.22	150	145
	13	300	24.3	500	3.6	300	0.22	150	148
	14	300	46.7	500	3.6	300	0.22	150	185
	15	300	46.7	500	3.6	300	0.22	150	186
	16	300	23.7	500	2.3	300	0.13	150	117
	17	300	23.7	500	2.3	300	0.13	150	115
	18	300	26.6	500	3.6	300	0.13	150	113
	19	300	26.6	500	3.6	300	0.13	150	129
	20	300	49.3	500	3.6	300	0.13	150	149
	21	300	49.3	500	3.6	300	0.13	150	137
	22	300	22.2	500	5.6	300	0.13	150	131
	23	300	22.2	500	5.6	300	0.13	150	151
	24	300	45.5	500	5.6	300	0.13	150	163
	25	300	45.5	500	5.6	300	0.13	150	164
	26	300	25.1	500	2.3	300	0.13	150	101
	27	300	25.1	500	2.3	300	0.13	150	113
	28	300	48.9	500	2.3	300	0.13	150	114
	29	300	48.9	500	2.3	300	0.13	150	128
	30	300	24.3	500	3.6	300	0.13	150	98
	31	300	24.3	500	3.6	300	0.13	150	122
	32	300	47.1	500	3.6	300	0.13	150	114
	33	300	47.1	500	3.6	300	0.13	150	150
	34	300	22.8	500	5.6	300	0.13	150	125
	35	300	22.8	500	5.6	300	0.13	150	134
	36	300	45.3	500	5.6	300	0.13	150	158
	37	300	45.3	500	5.6	300	0.13	150	175
	38	300	43.9	500	5.6	300	0.22	150	218
	39	300	36.1	500	5.6	300	0.22	150	206
	40	300	36.3	500	5.6	300	0.22	150	197
	41	300	34.1	500	5.6	300	0.22	150	183
	42	500	37.8	500	2.6	300	0.14	140	313
	43	500	37.8	500	2.6	300	0.14	140	366
	44	500	32.9	500	2.6	300	0.14	140	301
	45	500	32.9	500	2.6	300	0.14	140	329
Capon & de Cossio (1965)	46	251	13.2	400	3.08	250	0.1	250	59.5
	47	251	13.1	400	3.08	250	0.2	125	82
Kim (2000)	48	445	40.4	460	3.86	445	0.16	200	323
	49	445	36	460	3.86	445	0.21	150	411
	50	445	36	460	3.86	445	0.32	100	479

3.1 RC elements without shear reinforcements

According to NTC 2018 and Eurocode 2, the shear resistance of RC members without shear reinforcements (V_{Rd}^{wsr}), is evaluated by the formulation:

$$V_{Rd}^{wsr} = \left[\frac{0.18 \cdot k \cdot (100 \cdot \rho_l \cdot f_{ck})^{\frac{1}{3}}}{\gamma_c}\right] \cdot b \cdot d \quad (1)$$

where d is the effective depth of the cross section, b is the cross-section width, f_{ck} is the characteristic cylinder compression stress of concrete, ρ_l is the ratio $\frac{A_{sl}}{b \cdot d}$ with A_{sl} the longitudinal reinforcement area and finally γ_c is the safety factor for concrete resistance.

In Eq.1, the expression takes into account the concrete and steel bar interaction given by both the dowel effect and scale effect including the k scale factor. The dowel effect is significant for circular cross-sections and, as was previously discussed, is more effective than rectangular ones. This is due to the increased number of reinforcements along the cross-section perimeter than in a rectangular cross-section perimeter. However, the shear strength does not increase proportionally compared to the number of sub-vertical reinforcements.

Based on these considerations, in Eq.2 a simplification of Eq.1 is given, adapted specifically for circular cross-sections.

$$V_{Rd}^{wsr} = \alpha \cdot \left[(100 \cdot \rho_l \cdot f'_c)^{\frac{1}{3}} \cdot A_c\right] \quad (2)$$

where A_c is the circular cross-section area, the parameter ρ_l is redefined as the ratio $\frac{A_{sl}}{A_c}$. Finally, the α coefficient increases the shear strength in order to consider the more effective shear mechanism (i.e. in particular of the dowel effect) for circular cross-sections than rectangular ones.

In this research α coefficient was estimated by fitting experimental results reported in Table 1. The value assumed is equal to 0.293. According to the previous assumptions, in Eq.3 a formulation is given to assess the shear strength for RC without shear reinforcement elements.

$$V_{Rd}^{wsr} = 0.232 \cdot D^2 \cdot (100 \cdot \rho_l \cdot f'_c)^{\frac{1}{3}} \quad (3)$$

3.2 RC elements with shear reinforcements

The shear resistance of RC elements with reinforcements (V_{Rd}^{sr}) is defined starting from the formulation of Eq. 3, increasing it by the amplification coefficients $1 + \beta \cdot \rho_w$. The result is reported in Eq.4

$$V_R^{sr} = V_{Rd}^{wsr} \cdot (1 + \beta \cdot \rho_w) \quad (4)$$

where, ρ_w is the ratio $\frac{A_{sw}}{s \cdot D}$, A_{sw} is the stirrup area (i.e. the sum of two leg cross-section bar areas), s is the step of the stirrups and β is equal to 238 estimated by fitting experimental results reported in Table 2.

Eq.5 gives the final expression of the shear strength for RC elements with shear reinforcements.

$$V_R^{sr} = V_{Rd}^{wsr} \cdot (1 + 238 \cdot \rho_w) \quad (5)$$

4 MODEL VALIDATION

The shear strengths estimated by Eq.3 and Eq. 5, are compared with experimental values given by Tables 1 and 2. Fig. 3 and 4 show a plot of experimental values (abscises) against numerical values (ordinates), respectively for RC elements without (35 samples) and with (50 samples) shear reinforcements.

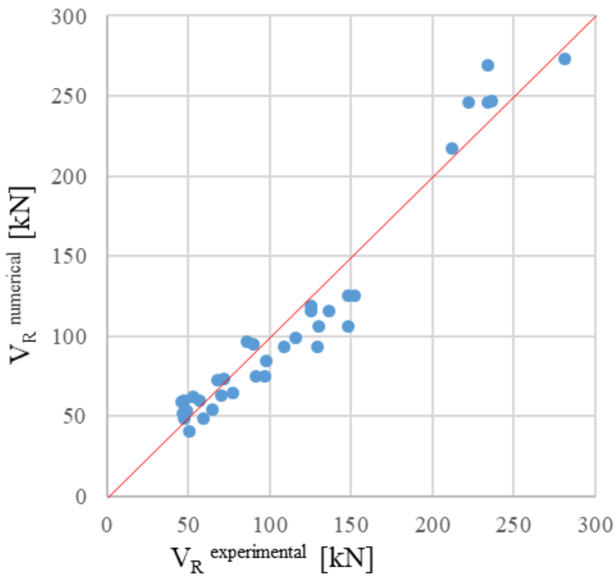

Figure 3. Comparison between numerical (Eq.3) and experimental values of shear strength for RC elements without shear reinforcements.

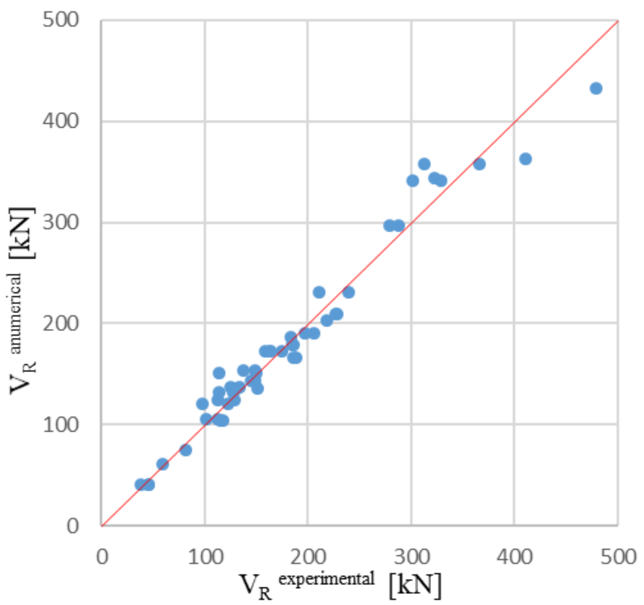

Figure 4. Comparison between numerical (Eq.5) and experimental values of shear strength for RC elements with shear reinforcements.

Tables 3 and 4 give the statistical indicators of the ratio between numerical and experimental values and show a very good result in particular for the (μ) mean value.

The proposed equations (i.e. Eq.3 and Eq.5) were compared with the results obtained by applying the formulation (Eq.6) suggested by Merta in 2006:

$$V_c = \xi \cdot [3.7 \cdot \rho_l + 0.18] \cdot k \cdot \sqrt{f_c'} \cdot 0.7 \cdot A_g \qquad (6)$$

where A_g is the cross-section area, ρ_l and f_c' were defined in Section 3. The k parameter depends on the ratio $\frac{a_s}{D}$ where a_s is the shear length (suggested equal to 1.25 for slender and 1 for short elements). Finally, ξ is expressed by Eq.7 and is representative of the scale effect.

$$\xi = \frac{1 + \sqrt{\frac{5.08}{d_a}}}{\sqrt{1 + \frac{d}{(25 \cdot d_a)}}} \qquad (7)$$

being d_a the maximum value of the aggregate diameter, whereas d is the effective depth of the cross section.

Table 3. Numerical (Eq.3)/experimental (Table 1) ratio and Merta (2006) /experimental (Table 1) ratio.

	by Eq.3	by Merta (2006)
Mean value (μ)	1.002	1.010
Standard deviation (σ)	0.154	0.129
Coefficient of variation (CoV)	0.154	0.128
Coefficient of determination (R^2)	0.943	0.958

In tables 3 the statistical indicators of the ratio between numerical and experimental values for Merta formulation (Eq. 6) are also reported.

With the purpose to demonstrate the robustness of the simplified formulation reported in Eq.5, the experimental data set given by Table 2 (RC elements with shear reinforcements) was also compared to numerical values obtained applying the formulation proposed by Fiore et al in 2014 (Eq. 8) and Merta in 2006 (Eq. 9).

As it was shortly discussed in Section 1, Fiore et al 2014 proposed five expressions that content numerical coefficients representative of the shear mechanism effect, except for the arc contribute. Eq.8 is the expression of Fiore et al that gives better statistical indicators compared with experimental values (i.e. Table 2) of others.

$$V_R^{Fiore} = 0.98243d\frac{A_{sh}}{s}f_{yw} + 0.086185Dd\sqrt{f_c'} \cdot \left(1 + 56.2\frac{A_{sl}}{Dd}\right) \qquad (8)$$

In Eq.8 A_{sl} and A_{sh} are the longitudinal bars and transversal stirrups cross-section areas respectively, f_{yw} is the yield stress of the stirrups steel.

Eq. 9 gives the shear strength proposed by Merta (2006) for RC elements with shear reinforcements.

$$V_s = A_{sh} \cdot f_{yw} \cdot (1.8 \cdot n_t + \lambda \cdot n_d + 1) \qquad (9)$$

In Eq. 9 n_t, $\lambda$and n_d are parametric values evaluated by longitudinal reinforcement ratio, axial load, section gross area, concrete compressive strength, cross section of the hoop, yield strength, shear-span, section's diameter, compression zone's depth and compressive struts inclination angle (i.e. truss model).

Table 4 summarizes the statistical indicators of the comparisons.

Table 4. Ratio between numerical (respectively given by Eq.5, Merta (2006) and Fiore et al (2014)) and experimental values of table 2.

	by Eq. 5	by Fiore et al.	by Merta
Mean value (μ)	1.000	1.017	1.041
Standard deviation (σ)	0.099	0.119	0.119
Coefficient of variation (CoV)	0.099	0.117	0.115
Coefficient of determination (R^2)	0.958	0.962	0.941

In order to enhance the comparison between values estimated by Eq.5 and Eq.8, a bigger sample was investigated. Totally, 520 different combinations of geometry (i.e. D, s, ρ_l and stirrups diameter d) and mechanical (i.e. f'_c, f_{yl} and f_{yw}) parameters were considered. Table 5 gives the expanded data set.

Table 5. Expanded geometrical and mechanic data set

D	cm	30-40-50-60-70-80-90-100-110-120-130-140-150
$f'c$	MPa	20-30
s	mm	100-200
ρ_l	%	0.3-0.6-1-2-4
f_{yl}	MPa	450
f_{yw}	MPa	450
d	mm	8

Fig.5 shows the comparison between the shear strength obtained by Eq.5 and by Eq.8 (Fiore et al, 2014) using the expanded data set given by Table 5. Table 6 gives the statistical indicators of the ratio illustrated in Fig.5.

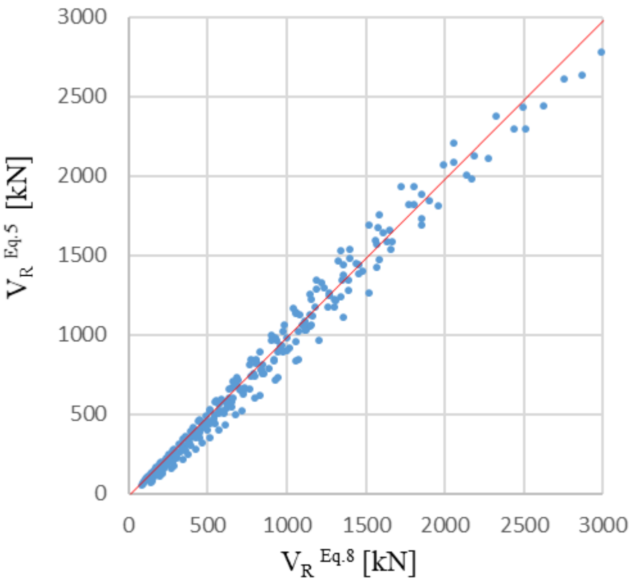

Figure 5. Shear strength numerical values given by Eq.5 against numerical values given by Eq.8.

The statistical indicators confirm the goodness of the proposed model (Eq.5) that gives satisfactory agreements with the more complicated formulation proposed by Fiore et al in 2014.

Table 6. Statistical indicators of the ratio illustrated in Fig.5

Mean value (μ)	*0.908*
Standard deviation (σ)	*0.124*
Coefficient of variation (CoV)	*0.136*
Coefficient of determination (R^2)	*0.983*

5 CONCLUSIONS

This paper discusses a new formulation to estimate the shear strength of RC elements with circular cross sections with and without shear reinforcements.

The proposed formulations are based on the significant contribution (improvement) given by the longitudinal bars uniformly distributed along the cross-section circumference (i.e. dowel effect) to shear strength. This contribution was estimated greater for circular than for rectangular or square cross-sections.

The successful aspect of the proposed theory allows for an estimation of the shear strength of elements, using amplification coefficients.

The formulations were calibrated on data sets resulting from experimental tests by literature. In order to measure the reliability of the proposed formulations, the coefficient of variation (Cov) and the coefficients of determination (R^2) were estimated, resulted equal to 16% and 94% and 10% and 96% for elements with and without shear reinforcements, respectively. These resulting values seem significantly satisfactory.

The values are better than the values obtained using others consolidate but more complicate formulations as proposed by Merta in 2006 and Fiore et al in 2014.

The robustness of the proposed formulation was also carried out on an expanded data sample (520 values).

In conclusion, the proposed method allows to evaluate the shear strength of circular RC cross-sections elements with and without shear reinforcement, by easy and user-friendly formulation.

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