Seismic assessment of a masonry church using rigid block limit analysis and continuous finite element modelling

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ABSTRACT

The aim of this study is to compare the results obtained from discrete and continuous modelling approaches in the assessment of the church of San Nicolò in Capodimonte (Genova, Italy) when subjected to horizontal loads induced by seismic actions. The discrete model is a three-dimensional model made of rigid blocks interacting by no-tension, frictional contact interfaces. The continuous model is a homogenized anisotropic finite element model, formulated in the framework of multi-surface plasticity and implemented in a FE code by means of a minimization algorithm. The comparison between the two modelling approaches was carried out in terms of failure mechanism, deformed configuration and lateral load capacity.

1 INTRODUCTION

The response of historic masonry structures under seismic events is usually affected by local failure mechanisms which typically involve both in-plane and out-of-plane collapse modes. The assessment of the seismic capacity related to these failure mechanisms can be carried out using different strategies. A variety of modelling approaches was proposed in past decades (Roca et al. 2010, de Felice et al 2017). Among those, discrete rigid block models and continuous finite element models represent two different alternatives that are commonly used for the prediction of local failure mechanism in historic masonry structures (de Felice and Giannini 2001), (D'Ayala and Speranza 2003), (Sorrentino et al. 2017), (Orduña and Lourenço 2005), (Sab et al. 2007), (Angelillo et al. 2010), (de Felice et al. 2010), (Marfia and Sacco 2012), (Portioli et al. 2014), (Roselli et al. 2018).

Considering the uncertainties that typically affect the structural knowledge of historic structures, numerical models should be as simple as possible and based on few mechanical parameters. In the meantime, the model should be refined enough to be able to capture the failure of masonry under tensile load, taking into account the anisotropic behavior, at least in terms of failure condition. Accordingly, the model should be validated on the basis of its capacity in reproducing the damage pattern under different boundary and loading conditions both, in-plane and out-of-plane.

In this perspective, the aim of this study is to compare the results obtained from a discrete rigid block model and a continuous finite element model, in order to evaluate potentialities and limitations of both approaches when applied to the analysis of the collapse mechanisms of the same case study.

The first model adopted is a discrete element model for three-dimensional limit analysis, which describes masonry as an assembly of rigid blocks interacting by no-tension, frictional contact interfaces (Portioli et al. 2014). The application of the proposed model to full-scale historic buildings still represent a challenging task, both for computational reasons and geometric modelling issues. On the one side, considering the large number of blocks which are comprised in a real structure, the solution procedure of the underlying optimization problem should be formulated to save CPU time. On the other side, efficient tools and algorithms for the generation of spatial rigid block models involving complex bond patterns should be developed to facilitate users' application and implementation (Portioli et al. 2014), (Cascini et al. 2018).

The second model adopted is a nonlinear three-dimensional finite element model (de Felice and Malena 2019) representing masonry wall as an elasto-plastic homogenized Love-Kirchhoff plate with an associated flow-rule. The model was formulated in the framework of multi-surface plasticity and implemented in a FE code by means of a minimization algorithm directly derived from the Haar-Karman principle. The model allows to carry out path-following analyses and provide the capacity curve derived from pushover analysis, expressing the variation of the collapse load multiplier as a function of a control point displacement.

The case study considered is the church of San Nicolò in Capodimonte (Genova, Italy). The comparison between the two modelling approaches was carried out in terms of failure mechanism, deformed configurations and capacity against increasing lateral loads.

1.1 *Rigid block model for limit analysis*

The adopted Rigid Block (RB) model represents historic masonry as an assemblage of rigid blocks interacting at the interfaces. A no-tension, frictional behaviour with infinite compressive strength is assumed at contact interfaces (Portioli et al. 2014). The calculation of the load factor α expressing the magnitude of lateral loads promoting the collapse is obtained from the solution of the static (i.e. lower bound) formulation of the limit analysis problem. The limit analysis problem is formulated in terms of second order cone programming, as follows:

$$\begin{aligned} \max \quad & \alpha \\ \textbf{s.t.} \quad & \mathbf{Ac}=\mathbf{f_D}+\alpha\mathbf{f_L} \\ & \mathbf{c} \in \mathbf{C} \end{aligned} \tag{1}$$

In the above optimization problem, the first constraint represents equilibrium conditions between internal static variables associated to contact interactions and external loads. The static variables are the internal forces acting at contact points, located at the vertexes of each interface. Those are collected in vector $\mathbf{c}$ and include the shear force components and the normal force at each contact point, acting along the tangent and normal direction to contact interface. External loads applied to the centroid of rigid block are expressed as the sum of dead loads $\boldsymbol{f_D}$ and live loads $\boldsymbol{f_L}$ multiplied by the collapse load factor α.

1.2 *Homogenized model for finite element analysis*

The adopted Finite Element (FE) model is an elastic perfectly-plastic homogenized plate model for the path-following finite element non-linear analysis of masonry walls subject to both in–plane and out-of-plane loads (de Buhan and de Felice 1997), (de Felice et al. 2010), (Amorosi et al. 2014), (de Felice and Malena 2019). According to the above referenced works, the elastic domain is assumed to coincide with the macroscopic strength condition defined in (Sab 2003), (Sab et al. 2007), derived by a homogenization procedure applied to a thin and periodic heterogeneous plate, made of 3D infinitely resistant blocks connected by Mohr-Coulomb interfaces obeying an associated flow rule. The model is formulated in the framework of infinitesimal multi-surface rate-independent plasticity. The macroscopic elastic domain assumes the following form:

$$\begin{aligned} E_f = \{ \mathbf{t}{:=}(\mathbf{N},\mathbf{M}) \mid f^i(\mathbf{N},\mathbf{M}) := \\ \mathbf{N{:}E}+\mathbf{M}{:}\boldsymbol{\chi} - c^i \le 0 \; \forall i \in [1..m] \} \end{aligned} \tag{2}$$

where **N, M** and $\mathbf{E}, \boldsymbol{\chi}$ are the vectors collecting the in-plane and the out-of-plane stresses and the strains, while $f^i(\mathbf{t})$ are the following *m=8* independent planes, intersecting in a non-smooth way:

$$\begin{aligned} f^{1-2} = \mu_b N_{11} + tg\phi N_{22} \pm (1+\mu_b\, tg\phi) N_{12} \\ -h\left(c + \frac{c\mu_b}{tg\phi}\right) \le 0 \end{aligned} \tag{3}$$

$$f^{3-4} = N_{22} \pm \frac{1}{tg\phi} N_{12} - \frac{hc}{tg\phi} \le 0 \tag{4}$$

$$f^{5-6} = N_{22} \pm \frac{2}{h} M_{22} - \frac{hc}{tg\phi} \le 0$$

(5) (5)

$$\begin{aligned} f^{7-8} = (p+q) N_{22} \pm \frac{2}{h} M_{11} - \frac{2}{h}(q-p) M_{22} \\ -\frac{h(p+q)c}{tg\phi} \le 0 \end{aligned} \tag{6}$$

with:

$$p = \frac{tg\phi}{\mu_b}\frac{b}{4h} \qquad q = \frac{tg\phi}{\mu_b}\sqrt{1+\left(\frac{b}{4h}\right)^2} \qquad (7)$$

The elastic domain depends explicitly on the cohesion c , the friction angle ϕ of the joints, the aspect ratio $\mu_b = \frac{2a}{b}$ of the blocks, where *a* is the height and *b* the width of the blocks, and on the thickness *h* of the plate. It is anisotropic as a consequence of the arrangement of the blocks within the assembly and unbounded in the direction of compression. For further details, the readers can refer to (de Felice et al. 2010), (de Felice and Malena 2019).

2 THE CASE STUDY

The numerical case study under investigation is the church of San Nicolò di Capodimonte (Camogli, Genova, Italy). A long single nave, of about 18 meters, is crossed by a transept 14 meters large (Fig. 1). A chapel and a bell tower are located in the transept next to the apses. The bell tower is about 17.0 m height while the nave is 14.0 m height.

A rigid block model and a finite element model were generated to investigate the structural behaviour of the construction under increasing lateral loads.

The discrete rigid block model of the church is made of 5.161 blocks and 49.976 contacts (Fig. 2) The average block size is 570 x 900 x 275 mm. The unit weight of masonry blocks for numerical simulation is 18.0 kN/m^3 and the friction coefficient at block interfaces is 0.6.

The 3D finite element model of the church is made of 64.993 shell elements for a total of 33.300 nodes. Boundary conditions were applied restraining horizontal and vertical displacements for all the nodes at the ground level. In accordance with the discrete model, the height *a* and the width *b* of the blocks were set equal to 275 and 570 mm, respectively, while the friction coefficient and cohesion were posed equal to 0.6 and to 0.0.

In a first step the building is analyzed under the vertical load deriving from the own weight. The total weight of the church is around 18.500 KN. Then, after the application of the self-weight, the seismic action is applied as a uniform distribution of lateral loads proportional to the weight, increasing up to the failure condition.

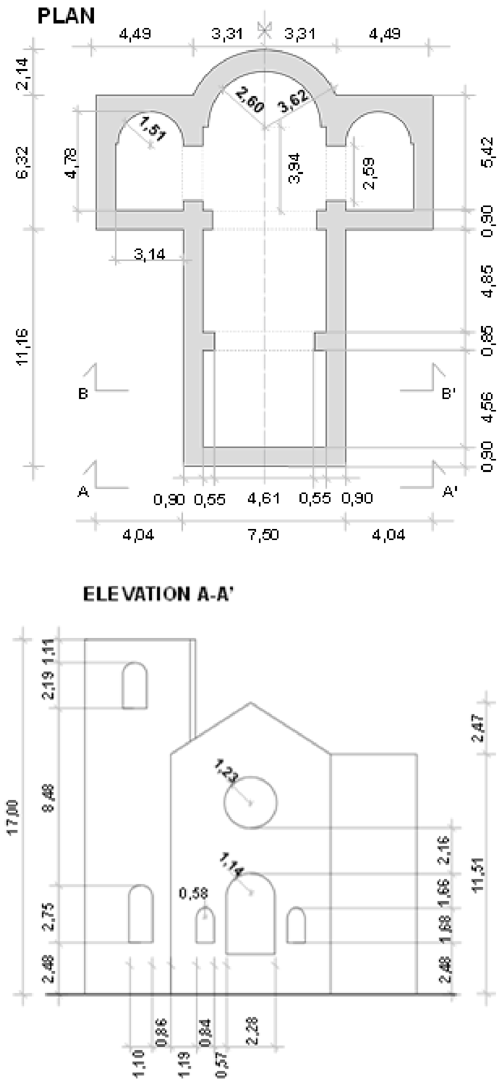

Figure 1. Geometry of the case study.

3 RESULTS OF THE COLLAPSE MECHANISM ANALYSIS

Two different directions were considered for the horizontal loads: the transverse direction (x-axis in Fig.2), parallel to the transept, and the longitudinal direction (y-axis in Fig.2) parallel to the nave. In Figs. 2 and 3, the crack pattern obtained with the rigid block model and the plastic strain pattern provided by the finite element model, are reported.

For transversal seismic loading (Figs. 2a, 3a), the collapse mechanism involves the in-plane failure of the main facade and of the triumphal arches, through the formation of diagonal cracks, while the longitudinal walls remain almost undamaged.

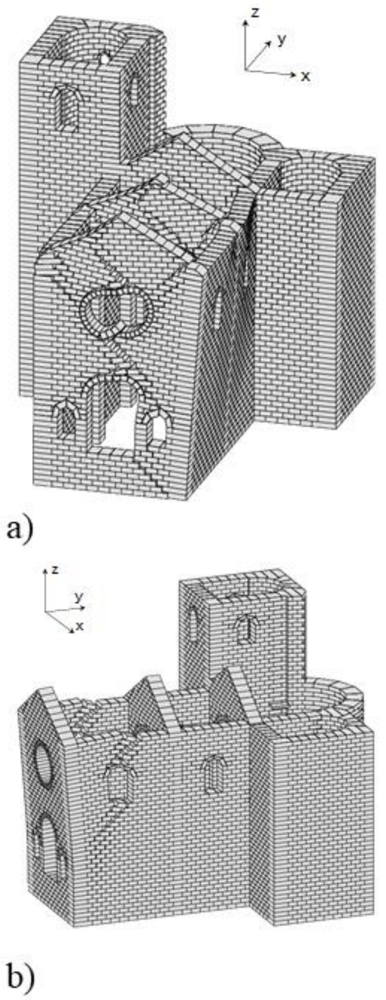

Figure 2. RB model of the church: crack pattern and corresponding failure mode for lateral load applied along the transversal x-axis (a) and longitudinal y-axis (b).

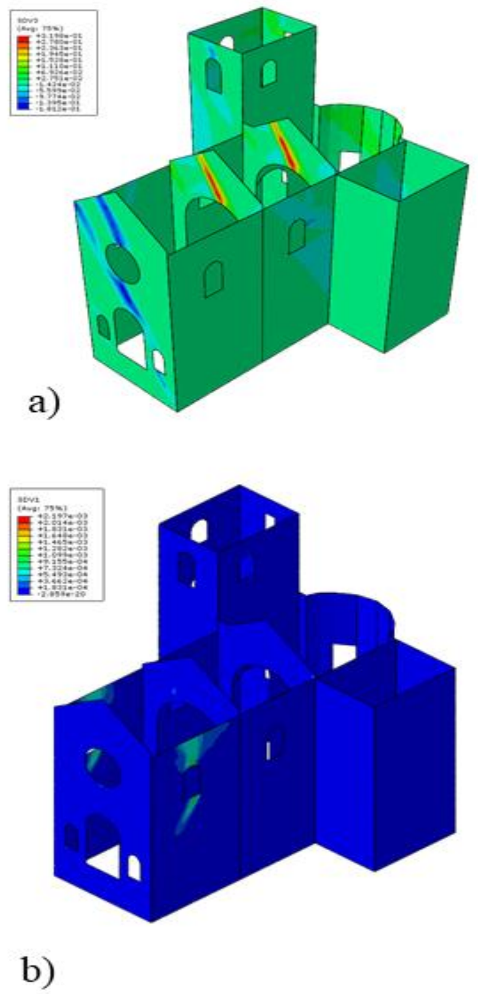

Figure 3. FE model of the church: plastic strain field at failure for lateral load applied along the transversal x-axis (a) and longitudinal y-axis (b).

For longitudinal seismic loading (Figs. 2b, 3b), both models predict the overturning of the main façade together with a portion of the sidewalls.

The failure load multipliers are 0.221 and 0.210 in x direction, and 0.237 and 0.229 in y direction, respectively for the associative solution of RB model and for the FE model. The non-associative formulation of the RB model returns 0.201 and 0.224 values for load multipliers, in the case of lateral loads in x and y directions. It is worth noting that the results obtained from RB and FE models are in a good agreement, both in terms of failure mode and collapse load multiplier, when the associative formulation of the RB model is considered. With reference to this last case, a discrepancy in the failure load multiplier of about 5.3% and 3.5%, is observed along x and y direction, respectively. For the RB model, it is shown that the difference of associative and non-associative collapse load multipliers is up to 10 percent.

In Figs. 4 and 5, the plot of the load multiplier vs. the displacement at the control point provided by the FE model is represented, for both transversal and longitudinal lateral loads. In both cases, the control point was located on the top of the façade, where the maximum displacement was achieved. The threshold represented by the failure load multiplier of RB analyses are represented as well.

A sensitivity analysis to the variation of the friction coefficient and block aspect ratio is finally carried out. For both models, a change in the friction coefficient directly affects the value of the collapse load multiplier, without significant changes in the collapse mechanism configuration. When changing the aspect ratio of the blocks a change in the inclination of the cracks and in the plastic strain localization is detected. For instance, by adopting blocks having the same height, but splitting in half their base, the crack pattern under transversal lateral load (Fig. 6) is characterized by an almost vertical crack above and below the rose windows, while in the previous case the crack exhibits a diagonal trend from the upper left corner up to the lower right corner of the façade (Figg. 2-3). The reduction in the collapse load factors due to the change in the shape of the blocks is equal to about 30.4 and 33.5 percent, for the FE and the RB model, respectively.

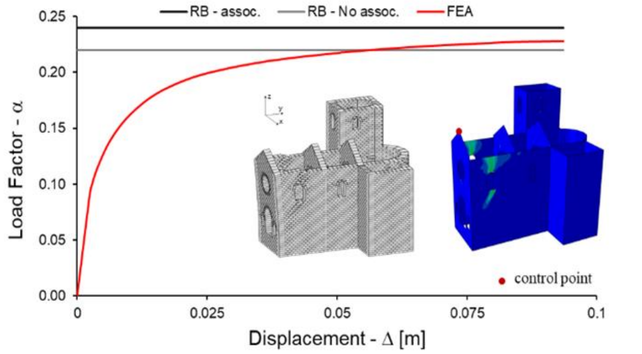

Figure 4. Comparison between RB and FE results in terms of load factor vs. lateral displacement, for horizontal loads in transversal (x-axis) direction.

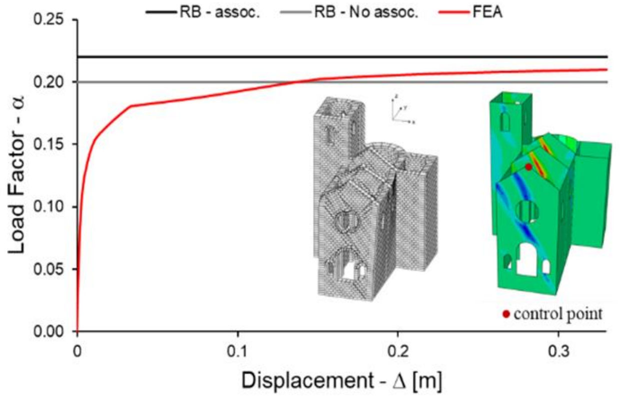

Figure 5. Comparison between RB and FE results in terms of load factor vs. lateral displacement, for horizontal loads in longitudinal (y-axis) direction.

4 CONCLUSIONS

A discrete rigid block model and a continuous finite element model of an ancient masonry church were generated in order to compare the outcomes of different modelling approaches when masonry is schematized as an assemblage of infinite resistant units interacting by means of Coulomb frictional interfaces.

The comparison between the numerical results of the two models was carried out in terms of failure modes and deformed configurations, as well as in terms of lateral loads promoting the collapse.

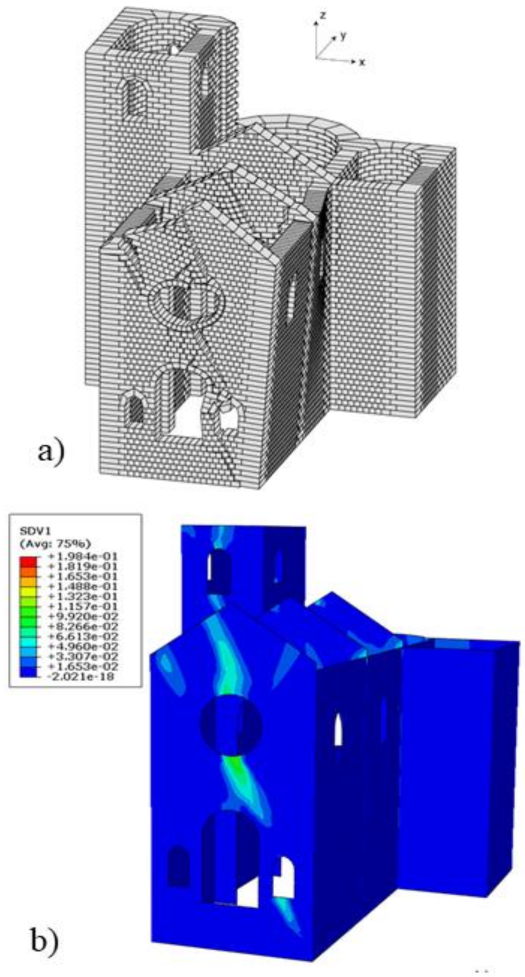

Figure 6. Failure pattern under transversal load: a) Cracks in the non-associative RB model (α_non-assoc. = 0.161) and b) plastic strain distribution predicted by the FE model for the reduced block size (α_c = 0.167).

A good agreement between the failure mechanisms was observed, confirming the ability of the discrete and the continuous models to capture the ultimate behavior under either in-plane or out-of-plane loads.

Both models are based on few mechanical parameters, namely the friction coefficient and masonry bond pattern. It should be noticed that more accurate models are available in the literature succeeding in a more accurate description of the nonlinear behavior of masonry. However, the use of such models generally requires many data inputs that are hardly available in the actual practice. In this perspective, the nonlinear models presented in this study might represent a good compromise among accuracy of results, necessary input data, and computational cost.

It is worth noting that in both approaches, the failure mechanisms are directly provided by the analysis and not a-priori defined, as it required in current seismic provisions. Accordingly, both numerical approaches represent promising tools for the analysis historic masonry buildings under earthquake loading.

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REFERENCES

Roca, P., Cervera, M., Gariup, G., Pela', L. 2010. Structural Analysis of Masonry Historical Constructions. Classical and Advanced Approaches. Archives Comput Methods Eng; **17**: 299-325.

de Felice, G., De Santis, S., Lourenco, P.B., Mendes, N. 2017. Methods and challenges for the seismic assessment of historic masonry structures. International Journal of Architectural Heritage; 11(1):143-160.

de Felice, G., and Giannini, R. 2001. Out-of-plane seismic resistance of masonry walls. Journal of Earthquake Engineering; **5** (2): 253–71. doi:10.1080/ 13632460109350394.

D'Ayala, D., and Speranza, E. 2003. Definition of collapse mechanisms and seismic vulnerability of historic masonry buildings. Earthquake Spectra; **19**(3): 479–509. doi:10.1193/ 1.1599896.

Sorrentino, L., D'Ayala, D., de Felice, G., Griffith, M.C., Lagomarsino, S., Magenes, G. 2017. Review of out-of-plane seismic assessment techniques applied to existing masonry buildings. International Journal of Architectural Heritage; **11**(1): 2-21.

Orduña, A., Lourenço, P.B. 2005. Three-dimensional limit analysis of rigid blocks assemblages. Part I: torsion failure on frictional joints and limit analysis formulation. Int. J. Solids Struct.; **42**(18-19): 5140-5160.

Sab, K. 2003. Yield design of thin periodic plates by a homogenization technique and an application to masonry walls. C. R. Mecanique; **331**(9): 641-646.

Sab, K., Cecchi, A., Dallot, J. 2007. Determination of the Overall Yield Strength Domain of Out-of-Plane Loaded Brick Masonry. International Journal for Multiscale Computational Engineering; **5**(2): 83-92.

de Felice, G. and Malena, M. 2019. Failure pattern prediction in masonry. Journal of Mechanics of Materials and Structures. Submitted for publication.

de Buhan, P., de Felice, G. 1997. A homogenization approach to the ultimate strength of brick masonry. J. Mech. Phys. Solids; **45**(7): 1085-1104.

Angelillo, M., Cardamone, L., Fortunato, A. 2010. A numerical model for masonry-like structures. Journal of Mechanics of Materials and Structures; **5**(4): 583-615.

de Felice, G., Amorosi, A., Malena, M. 2010. Elasto-plastic analysis of block structures throught a homogenization method. Int J Numer Analyt Methods Geomech; **34**(3): 221-247.

Marfia, S., Sacco, E. 2012. Multiscale damage contact-friction model for periodic masonry walls. Computer Methods in Applied Mechanics and Engineering; **205-208**(1): 189-203.

Portioli, F., Casapulla, C., Gilbert, M., Cascini, L. 2014. Limit analysis of 3D masonry block structures with non-associative frictional joints using cone programming. Computers and Structures; **143**: 108-121.

Roselli, I., Malena, M., Mongelli, M., Cavalagli, N., Gioffrè, M., De Canio, G., de Felice, G. 2018. Health assessment and ambient vibration testing of the "Ponte delle Torri" of Spoleto during the 2016–2017 Central Italy seismic sequence. Journal of Civil Structural Health Monitoring; **8**(2): 199-216.

Cascini, L., Gagliardo, R., Portioli, F. 2018. LiABlock_3D: A Software Tool for Collapse Mechanism Analysis of Historic Masonry Structures. International Journal of Architectural Heritage. doi: 10.1080/15583058.2018. 1509155.

Amorosi, A., Boldini, D., de Felice, G., Malena, M., Sebastianelli M. 2014. Tunnelling-induced deformation and damage on historical masonry structures. Géotechnique; **64**(2): 118-130.