Simplified model for the seismic check of masonry arch bridges with finite compression strength

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ABSTRACT
The mechanism method is used in this paper for the evaluation of the capacity of stone and masonry arch bridges under seismic loadings. The material is supposed to have a rigid-perfect plastic behaviour in compression and no tension strength. The arch is subject to the dead loads, due to its self-weight and the weight of the backfill, and to an horizontal seismic acceleration acting in the longitudinal direction, from the right to the left. This acceleration causes a horizontal inertial load acting on the entire arch due to the mass of the ring. Besides, the horizontal inertial actions due to the mass of the backfill, present from the two abutments to the arch profile, are considered only on left half of the arch, while the backfill is supposed to tend to separate from the arch at the right half. The main aspects of the structural behavior are pointed out and the influence of the various geometrical and loading parameters on the collapse acceleration value are investigated.

1 INTRODUCTION

The limit analysis is widely used for the assessment of stone arch bridge capacity. Numerical solutions based on the mechanism method were proposed by several authors and applied to study the seismic limit behaviour. The study consisted in finding out the collapse mechanism under dead - plus - vertical travelling loads and horizontal forces simulating the seismic actions and the horizontal acceleration needed to turn the structure into a mechanism.

Kooharian (1952) introduced the limit analysis of voussoir arches, based on the main hypothesis that stone arches fail by forming pin joints, as demonstrated by old but also by more recent experimental studies. As a result, the collapse must be viewed as a geometrical issue rather than a problem of strength of material.

Heyman (1966, 1969) proposed the well-known model of arch made of a set of rigid voussoirs laid dry without any mortar, with the assumptions of material with no tensile strength but infinite strength in compression and that sliding failure cannot occur. Heyman's theory was applied to study the limit behaviour and to find out the collapse mechanisms of stone arches under dead - plus - vertical live loads and to investigate, by means of a comprehensive numerical investigation, the influence of the various geometrical and loading parameters on the limit structural behaviour of stone arches (Clemente et al., 1995).

The mechanism model, has been also used to analyze the behaviour of a stone arch bridge under seismic actions. The dynamic behaviour of a circular arch was first analyzed by Oppenheim (1992), who established and discussed the failure conditions under a horizontal longitudinal acceleration. Clemente (1998) analyzed the dynamic behaviour of an arch without backfill under sinusoidal base acceleration and focused the attention on the importance of frequency content and amplitude of the input and of the initial conditions. Experimental studies on shaking table confirmed the hypothesis that stone arches fail by forming pin joints (Clemente et al., 1999).

De Lorenzis et al. (2007) addressed the impact condition for the circular arch, in the hypotheses that the hinge locations in the four link mechanism do not vary and they reflect when the motion is inverted. Furthermore, the model does not allow initial free hinge formation and assumes two hinges at springing points (Dejong et al., 2008). Actually, the four-hinge mechanism will hinge either only at one or at both springing points, depending on the value of the embracing angle and the thickness of the arch (Clemente, 1998), as confirmed also by using the variational method (Alexakis & Makris, 2014). The seismic behaviour of multispan masonry arch bridges was also analysed (De Felice et al., 2006), as well as the suitable retrofit techniques (Zampieri et al., 2015).

Actually, when the mechanism is put in action, the arch may return to its natural configuration, after one or more oscillations. In spite of that, from a technical point of view the comparison between the design peak ground acceleration at the site and the minimum acceleration necessary to turn the structure into a mechanism can be assumed as safety criterion. In this way, the safety check of a stone arch under seismic actions can be viewed as a static matter.

On the basis of these considerations, Clemente and Raithel (1998) carried out a comprehensive numerical investigation on the collapse mechanism of stone voussoir arches under horizontal longitudinal loadings simulating the seismic actions and the related load factor. The model of an arch with the Heyman's hypotheses was assumed, subject to its self-weight and backfill. Both the parabolic and circular shape were considered. Different models were assumed to simulate the structure - backfill interaction.

It is well known that the mechanism method presents very interesting advantages, such as the simplicity and the speed, but its application could be limited due to the limited validity of the assumed hypotheses. Among these the hypothesis of infinite compression strength.

As a matter of fact, the compression strength is not infinite, even though it could be very high. A relevant proposal was based on the definition of the thrust zone, which is of sufficient depth to carry the load at each cross-section (Harvey, 1988). Taylor and Mallinder (1993) make use of a parabolic stress-strain constitutive relationship derived from empirical observation to describe the local crushing of masonry at hinge development. Brencich and Di Francesco (2004) proposed an iterative procedure for the elastoplastic analysis of masonry arch structure with inelastic strains. Crisfield & Packham (1987) developed a numerical procedure to evaluate the collapse load for masonry arch bridges, using the mechanism method with finite compressive strength of the masonry.

In this paper the model of arch made of no-tension material and with a rigid-perfect plastic behaviour with finite strength in compression is considered. This model was already introduced and tested, by analysing the limit behaviour under vertical dead and travelling loads (Clemente & Saitta, 2017). In order to prescind from the position of the interface section and the size of the voussoirs, a very high number of voussoirs was assumed, simulating a continuous model. Furthermore, the case of seismic actions was considered, assuming that the arch was subjected also to a horizontal load proportional to the vertical load acting on it at each section (Saitta et al., 2016).

Herein the arch is considered subject to the dead loads, due to its self-weight and the weight of the backfill, and to an horizontal acceleration acting in the longitudinal direction from the right to the left. This acceleration causes a horizontal inertial load acting on the arch due to its mass. The horizontal inertial actions due to the mass of the backfill, present from the abutment to the arch profile, is considered only on the left half of the arch, while the backfill is supposed to tend to separate from the arch at its right half (Saitta et al., 2016).

The analysis is limited to the onset of motion and the horizontal acceleration necessary to turn the structure into a mechanism is evaluated. The successive dynamic phase is out of the scope of this paper. The influence of the various geometrical and loading parameters is analyzed, for the case of circular arch. The analysis is limited to the a plane arch. The contribution of spandrel walls and the structural contribution of the back-fill are not considered.

2 LIMIT ANALYSIS OF MASONRY ARCHES

2.1 *The plastic hinge and the collapse mechanism*

As already said, no tension resistance and a rigid-perfect plastic behaviour in compression, with finite strength f_u, were assumed for the material, (Clemente & Saitta, 2017). In this hypothesis, in a yielded section subject to an axial force N acting at a distance d from the edge, stresses are uniformly distributed along a depth $2d$ from the edge (Figure 1).

The yield domain of a rectangular cross-section in the plane $(\hat{e}, \hat{N})$ is:

$$\hat{e} = \pm\left(1 - \hat{N}\right)/2 \qquad (1)$$

where $\hat{e} = e/t$ and $\hat{N} = N/N_u$ (with $N_u = b \cdot t \cdot f_u$) are the non-dimensional eccentricity and the non-dimensional axial force, respectively. The couples $(\hat{e}, \hat{N})$ of the limit domain (Figure 2) correspond to limit states in which a portion of the cross-section, of depth $2d$ from the edge, is uniformly compressed with stress equal to f_u.

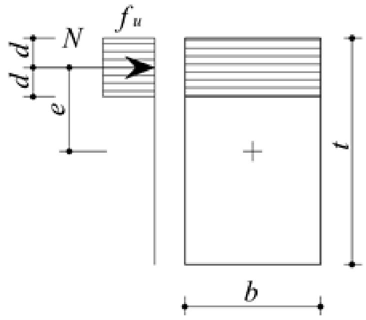

Figure 1. Design stress distribution at the limit state.

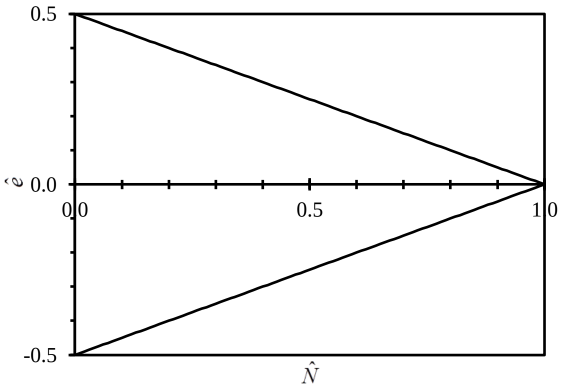

Figure 2. Limit domain in terms of $(\hat{e}, \hat{N})$.

The Heyman's model can be viewed as a limit case, in which $f_u \to \infty$ and $\hat{N} \to 0$, so that the two straight lines become parallel to $\hat{N}$ axis. The expression of the limit domain becomes $\hat{e} = \pm 1/2$.

In the hypothesis of rigid-plastic behaviour of the material with finite compression strength, the relative rotation centre, between two adjacent sections, on the point of collapse is coincident with the starting point of the stress diagram (Figure 3), i.e., at $2d$ from the edge. Therefore, the line of thrust does not pass through the hinge location and this implies some limitations to the possible mechanisms.

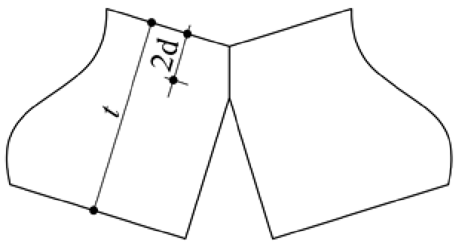

Figure 3. The plastic hinge.

Failure of an arch occurs when sufficient hinges (at least four hinges) form to turn the structure into a mechanism (Figure 4).

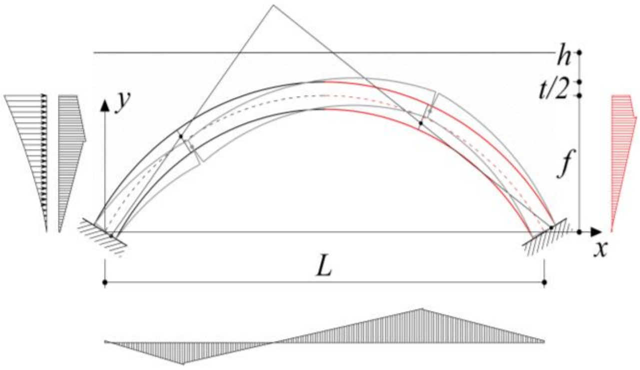

Figure 4. Collapse mechanism.

On the contrary to what happens when using the Heiman's model, the internal forces acting at the hinges contribute to the stability of the structure. Thanks to the hypotheses about the constitutive law of the material and the stress distribution, all the hinges form contemporary. The uniqueness theorem ensures that the solution exists and is unique, and so is the load factor. The safe theorem states that a structure is safe if an equilibrium solution can be found in which the couples $(\hat{e}, \hat{N})$ are always inside the limit domain.

It is important reminding that the thrust line of the safe theorem does not need to be the actual thrust line: every thrust line in equilibrium with external loads, and satisfying the limit condition, if any, can be chosen to check the structure. Moreover the actual stress distribution in the cross-section is not known. In fact, the assumption about the material constitutive relationship does not allow that, but the fact that the thrust line lies within the masonry, with stresses lower than f_u, ensures that there are only compressive actions, which can be transmitted from each section to the next (Clemente et al., 1995).

2.2 *The collapse horizontal acceleration*

For any span L, assuming the non-dimensional coordinates of the arch centreline:

$$\hat{x} = x/L \;,\; \hat{y}(\hat{x}) = y(x)/L \tag{2}$$

the geometrical characteristics of the arch are individualized by the sag ratio, the thickness ratio function and the fill depth ratio above the extrados at the crown, respectively:

$$\hat{f} = f/L \qquad \hat{t}(\hat{x}) = t(x)/L \qquad \hat{h} = h/L \tag{3}$$

The width b of the deck is usually assumed unitary for a plane modelling. If γ_w is the weight per unit volume of the structural material that constitutes the ring, its compression strength can be defined by the non-dimensional parameter (Clemente and Saitta, 2017):

$$\sigma = f_u/\gamma_w L \tag{4}$$

The total dead load w is given by the summation of:

- the arch self-weight w_w, which varies along the span even if the thickness is constant,
- the backfill weight w_b, also variable along the span, which depends also on the backfill weight per unit volume γ_b.

The horizontal load depends on the model assumed for the structure-backfill interaction. In the following, the arch is subjected to an horizontal acceleration acting in the longitudinal direction from the right to the left. This acceleration causes a horizontal inertial load acting on the entire arch due to the mass of the ring.

The horizontal inertial actions due to the mass of the backfill, present from the abutment to the arch profile, is considered only on the left half of the arch, while the backfill is supposed to tend to separate from the arch at the right half. Then, the horizontal seismic loading is:

$$p_h(x) = \left[w_w(x) + \gamma_b \tan\alpha\left(x - t/2\sin\alpha\right)\right]\cdot a_g/g \tag{5}$$

Let us consider the masonry arch in Figure 4 and suppose that an equilibrium solution under the vertical dead loads can be found, in which the points representing the stress states on the plane ($\hat{e}, \hat{N}$) are always within the limit domain of each cross-section. When the seismic loads are put in action and are increased from zero to the collapse value, the line of thrust changes and then at least four hinges form.

The collapse mechanism and the corresponding horizontal acceleration can be found by using the usual iteration procedure in which the equilibrium equation is written by means of the principle of virtual works (Clemente et al., 1995). For any assigned mechanism, the virtual works of the vertical loads w and the reference horizontal load p_h, obtained assuming $\hat{a}_g = a_g/g = 1$ in eq. (5), can be written as follows, using the non-dimensional expressions of the loads, respectively:

$$\begin{aligned} L_w &= \gamma_w b L^3 \cdot \int_0^1 \left[\hat{w}_w(\hat{x}) + \hat{w}_b(\hat{x})\right]\cdot \eta(\hat{x})\, d\hat{x} \\ L_h &= \gamma_w b L^3 \int_0^1 \left[\hat{w}_w(\hat{x}) + \hat{w}_b(\hat{x})\right]\cdot \hat{\xi}(\hat{x})\, d\hat{x} \end{aligned} \tag{6}$$

The function $\eta(\hat{x})$ and $\hat{\xi}(\hat{x})$ are the vertical and horizontal components of the virtual displacements, respectively. If $\Delta\varphi_i$ are the relative rotations at the n hinges and $\hat{N}_i = 2\sigma\hat{d}_i$ are the corresponding axial forces, then the internal work can be written:

$$L_i = 2\sum_1^n f_u d_i^2 \Delta\varphi_i = 2\gamma_w b L^3 \sum_1^n \sigma \hat{d}_i^2 \Delta\varphi_i \tag{7}$$

It is interesting to observe that when the compression strength increases indefinitely, then the internal work tends to zero. If $\hat{a}_g$ is the horizontal acceleration that turns the structure into a mechanism, the equilibrium equation can be written:

$$L_w + \hat{a}_g L_h = L_i \tag{8}$$

Eq. (8) gives the kinematically admissible horizontal acceleration $\hat{a}_g$ and so the load intensity associated with the assumed mechanism, from which the external reactions and the line of thrust can be found. The acceleration value $\hat{a}_g$ is the collapse acceleration only if the associated line of thrust satisfies everywhere the relation $|\hat{e}| \le (1-\hat{N})/2$, with the equality at a sufficient number of sections to turn the arch into a mechanism. If it is not, the procedure must be continued and in the next step the hinges must be moved to the section with the maximum exceedances.

3 LIMIT BEHAVIOUR OF A CIRCULAR ARCH UNDER SEISMIC LOADING

The geometry of a circular arch can be defined by means of the angle of embrace β and the radius R. They are related to span length L by the relationship $R = L/2\cdot\sin(\beta/2)$.

In the following, the value $\beta = 125°$ (for which it is $R = 0.564\cdot L$) is considered and the thickness is referred to the radius R. Furthermore, an average value was assumed for the backfill weight per unit volume. It was expressed as a ratio of the weight per unit volume of the arch, $\gamma = \gamma_b/\gamma_w = 0.5$. Finally two values of the material parameter σ were considered, equal to 10 and 30, respectively, and two values of the backfill depth $\hat{h}$, equal to 0 and 0.03, respectively.

The already described model was considered for the backfill – arch interaction, in which only the left side of the arch is subject to the inertial force due to the horizontal strips of the backfill acting on it.

In Figure 5 the collapse horizontal acceleration $\hat{a}_g$ is plotted versus *t/R*, for the two values of σ (10 and 30, respectively) and two values of $\hat{h}$ (0 and 0.03, respectively). The values of the acceleration, which turns the arch into a mechanism, increases almost linearly with *t/R*. In Figures 6 the couples $(\hat{e}, \hat{N})$ of the eccentricity and the axial force along the arch are plotted with the limit domain, for different values of *t/R* ranging from 0.03 to 0.15, and for two values of σ (10 in Figure 6a and 30 in Figure 6b, respectively).

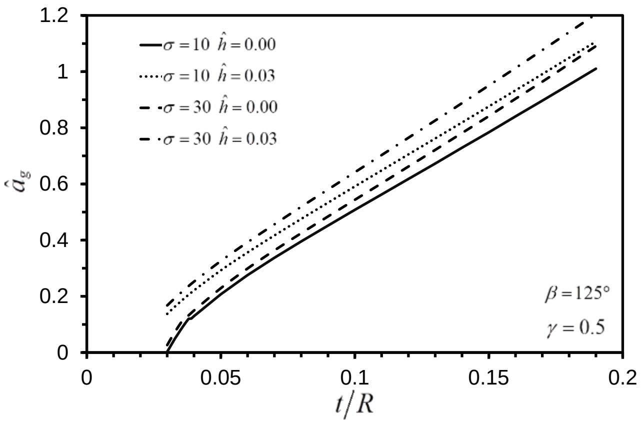

Figure 5. Collapse values of the acceleration $\hat{a}_g$ versus t/R, for β=125° and γ =0.5.

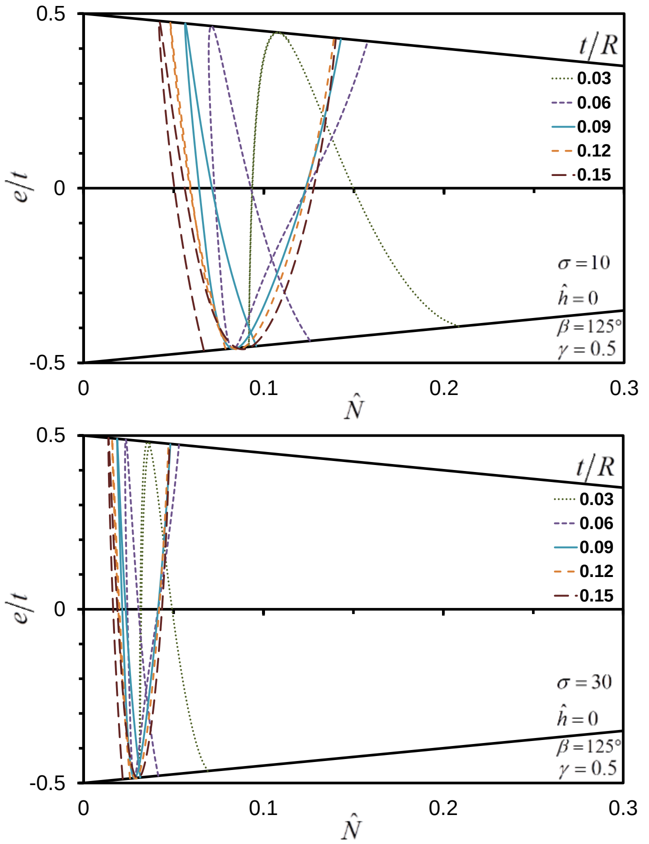

Figure 6. Couples of values of $(\hat{e}, \hat{N})$ at collapse, for different values of σ and $\hat{h}$, for β=125° and γ =0.5.

The larger σ the lower the value of the non-dimensional axial force, approaching zero when the strength of material goes to infinity, in accordance to the Heyman's model.

Figures 7 show the hinge position versus *t/R* for the two values of σ (10 and 30, respectively) and $\hat{h}$ = 0. It is apparent that, for the lower value of the fill height and for very low values of *t/R* (between 0.03 and 0.04) hinge H4 does not form at the right springing, $\hat{x} = 1$, but at a lower abscissa. Also relevant changes of the position of H3 can be observed for low values of *t/R*, for all the considered cases.

The values 2*d/t* of the plastic portions of the cross-sections at hinges versus *t/R* are plotted in Figures 8, for two values of σ (10 and 30, respectively) and $\hat{h}$ = 0. The influence of the fill height is evident. Furthermore, the movement of hinge H4, pointed out for very low values of t/R and $\hat{h}$ = 0, leads to a reduction of the plastic depth at that hinge.

The collapse mechanisms obtained for values of *t/R* in the range 0.03-0.04, with a step of 0.002, are shown in Figure 9, for σ = 10 and $\hat{h}$ = 0. It is worth noting that the position of the hinge H4 approaches for a lower thickness value in the case of rigid material. Finally, in Figure 10 the collapse horizontal acceleration $\hat{a}_g$ is plotted versus the rise $\hat{f}$ for two values of σ (10 and 30, respectively) and two values of $\hat{h}$ (0 and 0.03, respectively).

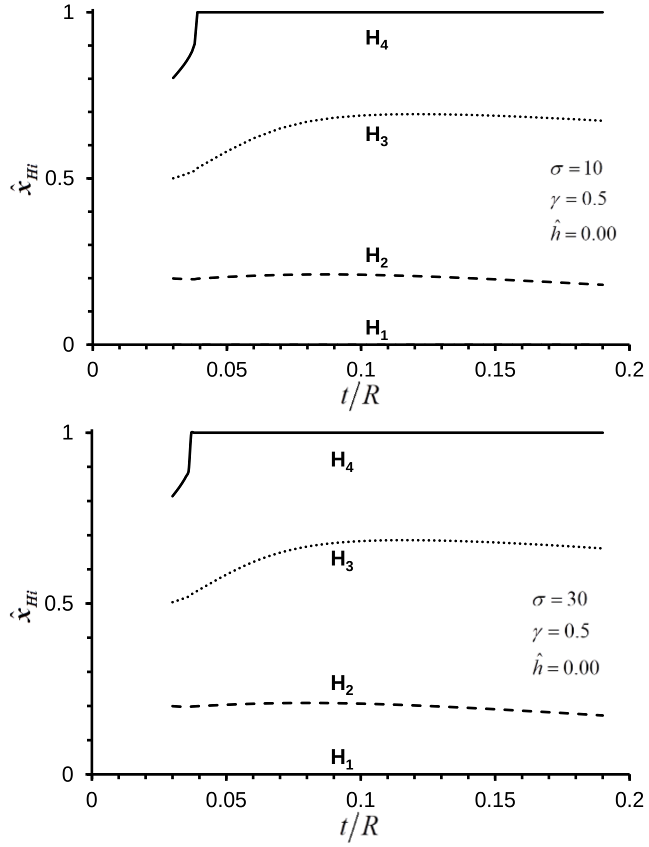

Figure 7. Plastic hinge locations for different values of σ (10 and 30), for $\hat{h}$ = 0, β = 125° and γ = 0.5.

The range of variability of $\hat{f}$ corresponds to values of β between 50° and 125°. As already observed in previous papers, the resistance of the arch increases when the sag ratio decreases.

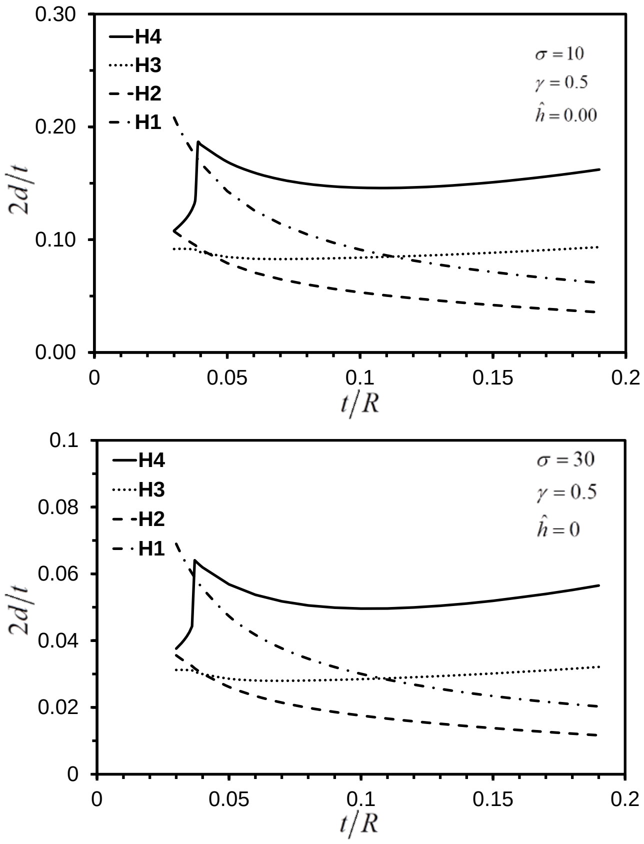

Figure 8. Plastic portions of the cross-sections at hinges for two values of σ (10 and 30), with $\hat{h}$ = 0, β = 125° and γ = 0.5.

4 CONCLUSIONS

In this paper the seismic behaviour of arch bridges made of no tension materials with finite compression strength has been analyzed. The hypothesis of material with rigid-perfect plastic behaviour in compression and no tension strength allows to interpret the effective behaviour of masonry, whose strength in compression is limited and could be quite low. In fact, hinges cannot form at one free edge of the arch cross-sections (this is almost true for very high material strength only), but a finite portion of the cross section will be yielded. The extension of this portion is obviously related to the masonry strength. In the proposed model the internal work assumes a great importance in the equilibrium condition, especially for masonry with low strength. The vertical loads, which include the ring self-weight and the backfill, were assumed to be fixed. The variable horizontal seismic loading was related to the inertial forces due to the arch ring mass plus that of the horizontal strips of backfill. A rigid-perfect plastic model was used for the material. The structural behaviour was investigated by numerical analysis based on a non-dimensional formulation.

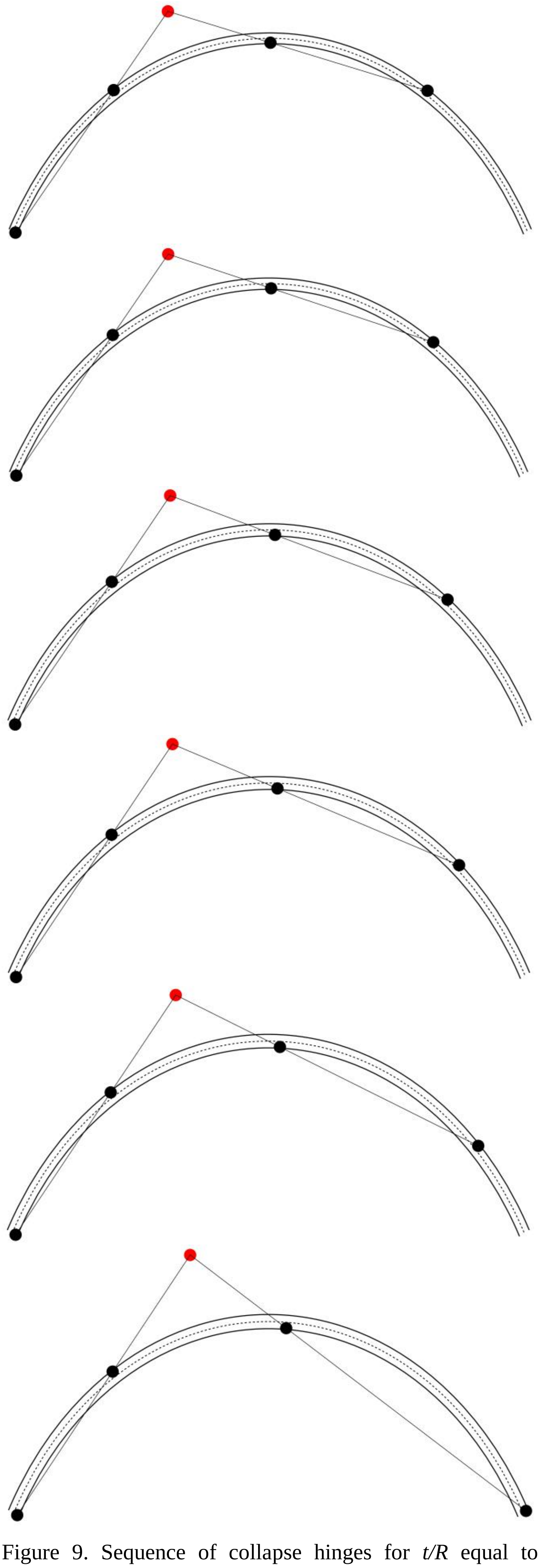

Figure 9. Sequence of collapse hinges for *t/R* equal to 0.030, 0.032, 0.034, 0.036, 0.038 and 0.040, respectively, with σ =10, $\hat{h}$ =0, γ =0.5 and β =125°.

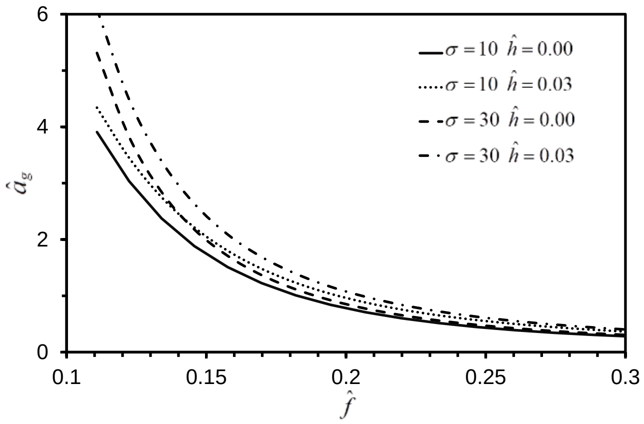

Figure 10. Collapse acceleration versus $\hat{f}$ for γ =0.5 and for two values of σ (10 and 30) and two values of $\hat{h}$ (0 and 0.03).

In general, the collapse horizontal acceleration increases with the thickness but presents a significant lowering when the strength of the material decreases. The presence of the fill influences very much the position of the hinges, especially for low values of the thickness. With reference to other horizontal loads, major differences in hinge positions are observed for low values of the thickness (Saitta et al., 2016). Further investigations are needed to highlight differences for a wider range of the various parameters.

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