



## Seismic design of high-speed rail viaducts

Lucio Chiodi<sup>a</sup>, Riccardo Strosio<sup>b</sup>

<sup>a</sup> Mott MacDonald, 8-10 Sydenham Road, Croydon, London, UK

<sup>b</sup> Tony Gee and Partners, 140 High St, Esher, Surrey, UK

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### ABSTRACT

High-speed railway lines are currently being planned, designed and constructed in several countries, most of them located in areas where important seismic risk is present. High speed railways require demanding alignment parameters, which coupled with the terrain profiles, result in a large number of bridge structures and viaducts to be provided along the rail corridor. At the same time, the stringent performance requirements imposed by the operation of high-speed trains – in particular the limitations on structures' displacements and movements - lead directly towards stiffer and heavier structural configurations which adversely impact the response in seismic conditions. The complexity of designing the high-speed rail infrastructure is therefore further complicated when seismic conditions have concurrently to be considered. The paper deals with the design considerations for high speed rail viaducts taking into account seismic excitation, illustrating the conceptual design and a design strategy to ensure control of the seismic response.

### 1 INTRODUCTION

High-Speed Rail (HSR) lines are extending rapidly worldwide as a competitive mode of transport alternative to aviation and road in the medium distance range and with greater passenger capacity. Such a diffusion covers many areas affected by significant earthquake activity

Design requirements for HSR lines are restrictive: large horizontal radii, small gradients, limited allowable settlements of earthwork embankments. The combination of these constraints with varied terrain topography and potential presence of flood plains often result in a significant proportion of the alignment being

elevated on viaducts for extended tracts (Fig. 1). The performance requirements for the elevated structures supporting the line are equally demanding as movements and rotations of the tracks must be severely restricted, leading to the provision of stiff and heavy structures, characteristics unfavourable for an efficient seismic design.

The strict criteria on limit displacements of the superstructure to safely operate the track and the commercial requirement to keep the line in operation after a seismic event, dictate the performance requirements for the structures under the frequent, service earthquake, without incurring damage; under the extreme event, where collapse must be prevented and damage be such that it can

be repaired. The seismic design of high-speed rail viaducts thus becomes a problem of control of the seismic response of the system, through provision of dissipation and isolation, design strategy defined on a case-by-case basis, following the paradigm of performance-based design: the structure designed to provide the required response to the corresponding demand level it is subjected to.



Figure 1. High-speed rail viaduct (Taiwan HSR)

## 2 HIGH-SPEED RAIL VIADUCTS

### 2.1 *Actions on structures and performance criteria*

HSR bridges and viaducts present specific characteristics related to the functional requirements of the railway.

Permanent loads - deck self-weight and superimposed loads - can be in excess of 400 kN/m; vertical design live loads are 80 kN/m per track, increased by an impact factor to take account of dynamic amplification effects. Horizontal actions due to braking and traction can reach maximum values of 6000 kN and 1000 kN respectively, considered acting simultaneously on a twin track, while the longitudinal action transferred by the continuous welded rail due to thermal effects and linear deck variations (creep and shrinkage) can reach values of similar magnitude. On curved viaducts, centrifugal forces can become significant.

Limits on live load span deflection and joint rotation are enforced to control the ride quality of the track at high operational speeds. Limits on twist of the track also apply, in particular at crossovers and turnouts. Vertical accelerations must be limited to ensure track stability and

passenger comfort. Relative displacements at expansion joints require stiff substructures, and in-plane rotation limits demand high transverse pier bending stiffness.

All these performance requirements result in structures, deck and substructures, generally stiffer and heavier than those of conventional railway bridges. These characteristics are in opposition to the requirements of the seismic design, resulting in structures with short natural periods of vibration and therefore high seismic force demands.

### 2.2 *Continuous welded rails on HSR*

Continuously welded rails are adopted for HSR tracks, as it is preferable to avoid rail joints, which are expensive and constitute a maintenance liability. Without rail joints, HSR bridges are limited to between 60m and 90m long, depending on the form of deck construction (steel or concrete, respectively), to avoid excessive stress build-up in the continuous rail. Relative longitudinal displacement at joints between adjacent bridge decks is limited to 5mm under braking and traction.

For bridges longer than 90m, long continuous viaducts, rail joints must be provided. In this case, the viaduct come in two sections about 36m apart, the bridge deck is joined at the same two locations, separated by a simply supported span. Such a joint will cater for an expansion length of about 2 x 400m. Relative longitudinal displacements under traction and braking is then limited to 30mm.

## 3 SEISMIC DESIGN OF HSR BRIDGES

### 3.1 *Performance requirements*

A two-level performance-based approach is usually adopted for the seismic design of HSR bridges: this include an operational seismic event, equivalent to a serviceability limit state, and an extreme seismic event, equivalent to an ultimate limit state. The operational seismic event has a short return period, typically 50 years with design ground acceleration of about a third of the extreme event. Under the operational event, the structures are designed to remain fully serviceable and track displacements must remain within the allowable limits. The extreme seismic event has generally long return periods, of approximately 950 years and more, where collapse must be avoided, and repairable damage is accepted with a capacity

design approach adopted. The rationale is to ensure safe operation during low return period events and avoidance of collapse for events with low probability of occurrence during the design life of the system.

Limit longitudinal displacements at expansion joints are of the order of 25mm for the operational earthquake and 100mm and more for the extreme event, to avoid clash between adjacent decks or between deck and abutment, or prevent loss of bearing support.

### 3.2 Conceptual seismic design

For the structure, a seismic event represents a demand to accommodate imposed dynamic displacements, particularly in the horizontal direction. The main decision in the conceptual seismic design of viaducts relates to how accommodate the horizontal displacements of the deck with respect to the supports.

The requirement to remain fully operational after the service design earthquake, ensuring an elastic response without structural damage, leads the choice of articulation to restrain the deck at the top of one or more piers near the stiffness centroid of a viaduct with continuous deck, the piers working as vertical cantilevers accommodating the seismic displacements by bending. In zones with low seismicity, monolithic connection with the deck of the taller, more flexible piers could be considered, provided the no damage requirement can be met for the corresponding service performance level.

In the transverse direction the superstructure is restrained at each pier support, regardless whether it is a continuous deck or simply supported multi-spans, either through guided bearings or, with available room at the top of the pier and expected high inertia forces – result of the seismicity level and/or length of the spans – through the provision of shear keys.

### 3.3 Multi-span simply supported viaducts

Simply supported multi-span viaducts have been widely employed in HSR applications [e.g., Chiodi and Strocio 2006 a, b] for their advantage to separate seismic design, as well as rheology and thermal effects, from rail-structure interaction, allowing the adoption of continuous welded rails. The drawback is reduced seismic structural efficiency, with respect to an equivalent continuous superstructure, and increased bearing

and seismic devices costs and maintenance. In viaducts with tall piers, out-of-phase movements between piers could take place, with increased risks of deck unseating and stresses in the rails. For long viaducts, such issues may be exacerbated by possible out-of-phase ground movements along the viaduct length. To control relative movements, some degree of restraint should then be installed, which would introduce an interaction between adjacent spans.

Fig. 2 and 3 illustrate the application of a multi-span bearing and restraints arrangement used in the Taiwan HSR (Chiodi and Strocio 2006 a, b). Transverse seismic forces are transferred through the shear keys. Longitudinal earthquake loads are transmitted directly to the shear keys through the elastomeric bearing pads in one direction and pretensioned tie bars into the shear key in the other. To ensure effective restraint of the deck during train traction/braking and to limit any displacement of the tie-bar during service earthquake, the tie bars are pre-tensioned to the strain/force corresponding to these effects.

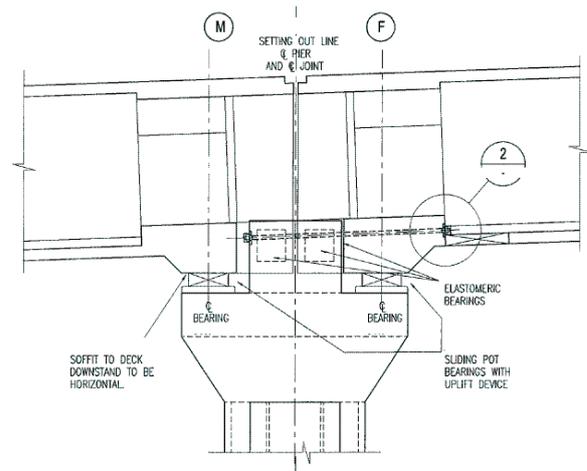


Figure 2. Longitudinal section of deck supports and restraint

Considering a bridge crossing a V-shaped valley which would require tall piers, the articulation with a series of simply supported spans enables to design out the need for rail movement joints. Horizontal forces due to earthquake, thermal effects and rail actions are shared among the supports. However, the pier stiffness required to control spans relative movements at each structural joint becomes excessive and uneconomical as the height of the piers increases.

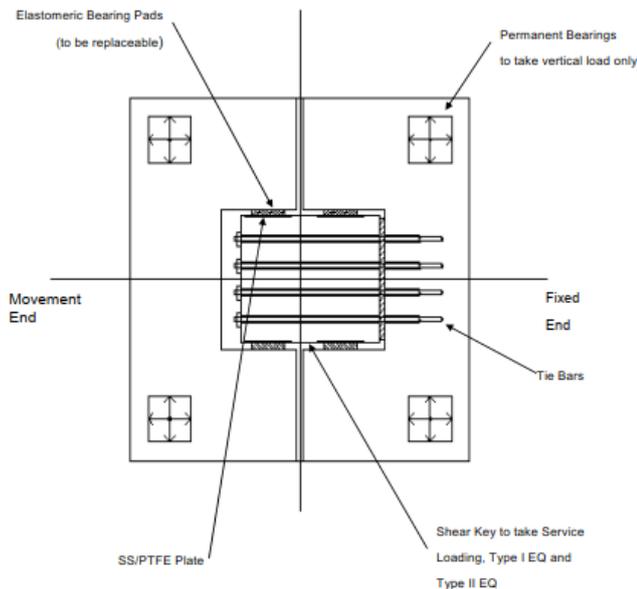


Figure 3. Schematic plan of deck supports and restraint

### 3.4 Continuous deck viaducts

The choice of the superstructure articulation, together with the placement of the seismic devices, have primary influence on the magnitude and distribution of forces the structure will face during an earthquake.

Compatibly with the performance requirements of the railway, continuous superstructures with the provision of minimum number of structural movement joints may represent the best compromise design solution. Several viaduct configurations can be envisaged in practice and some of the merits of the most common types are discussed in the following.

One of the most widespread schemes presents regular spans, low-medium piers' height, Fig. 4 [Cascales Fernández et al., 2017]. The substructures, due to the supports height and concrete cross-section, are usually stiff to horizontal movement and may require installing seismic isolation at the deck interface to reduce the magnitude of the seismic forces entering the substructures. The neutral point of longitudinal displacement with respect to temperature and rheology effects (creep and shrinkage) is located around the centre of the viaduct length. Elastomeric-based type bearings can be provided above the central piers, on the appropriate number of piers depending on seismicity level, piers' stiffness and viaduct expansion length, while the remaining supports are on sliding bearings to allow the increase of amplitude of movements and linear deck variations away from the centre. The central piers have also the function of providing

self-restoring action of the deck in case of earthquake. Self-restoring is the elastic capability of the structure to return to its initial position after the event, avoiding the build-up of residual permanent displacements and loss of accuracy in the prediction of maximum movements of the isolation system.

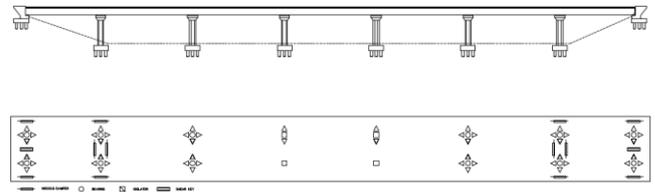


Figure 4. Constant height viaduct configuration

Longitudinal viscous dampers are provided at the required number of piers with guided bearings (i.e. where relative movement between deck and pier is permitted) to enhance the energy dissipation. Longitudinal seismic connection only at abutments through viscous dampers may be assessed and adopted for viaducts with shorter total length. In the transverse direction, there could be the need to isolate the substructure at the fixed piers to ensure an elastic response, together with the provision of transverse viscous dampers at selected location along the viaduct length to limit lateral deck displacements. The provision of shear keys, or guided bearings, to resist lateral movements may result in a stiffer dynamic response of the pier and should be assessed on a case by case basis.

The concept seismic design of a similar viaduct configuration as the previous, but with taller piers, slenderness greater than about 15, Fig. 5, can exploit the natural flexibility of the piers, providing fixed bearings at the central and taller piers, reducing second-order effects, allowing longitudinal movements over the shorter piers and at the abutments. The high flexibility of the piers avoids the built-up of restrained stresses due to deck linear variations (rheology and temperature), provides a degree of isolation, and the function of self-restoring the deck in case of seismic event. To control displacements of the superstructure within the limits, and control stresses in the fixed piers, viscous dampers capable to dissipate energy can then be provided over the shorter piers and at the abutments. To avoid overloading the shorter, stiffer piers and the abutments, and achieve a more uniform distribution of the seismic forces, the deck can be restrained in the transverse direction over the shorter piers with viscous dampers. As the tall

deck configuration is susceptible to wind loads and, if the horizontal alignment is curved, subject to centrifugal forces, prestressed viscous dampers may be adopted in the transverse direction over the shorter piers, with the function to act as shear keys to restrain wind and centrifugal forces, and as viscous dampers with energy dissipation in case of earthquake. In summary, over the shorter piers multi-directional bearings are provided, with the transverse prestressed viscous dampers providing the lateral restraint, while in the taller piers, provided of fixed bearings, stress levels and lateral displacements are restrained by their natural flexibility.

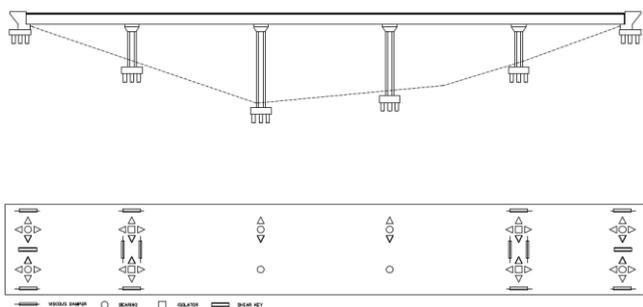


Figure 5. Varying height viaduct configuration

When dealing with valley-shaped viaduct configurations, Fig. 6, similar design concepts are followed. The objective is to distribute the seismic forces among the piers, while controlling displacements in both the principal directions, dissipating energy in the process. Fixed bearings are installed over the central and taller piers, isolation devices provided over the shorter piers, while dissipative viscous dampers provided over the stiffer piers and at the abutments.

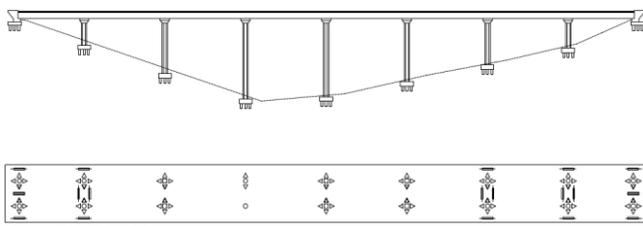


Figure 6. Valley-shaped viaduct with tall piers

Considering an irregular configuration with short piers, Fig. 7, the irregularity of the piers' heights makes it difficult to locate the centroid of the stiffness around the middle of the viaduct length, as the central pier is also the stiffer one. A solution could be to provide pendular bearings over all supports, longitudinal viscous dampers

over all piers, transverse viscous dampers over the piers adjacent the abutments, and laterally restrain the movement of the superstructure at the abutments [Santamaria Caballero et al., 2013]. In the longitudinal direction, the deck is floating, without fixed bearings, the restraint provided by the viscous dampers.

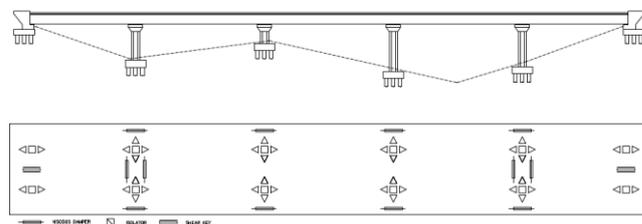


Figure 7. Irregular viaduct configuration with short piers

Provision of shock transmission units (STU) may not be always suitable in HSR applications, except perhaps in situations where some undesirable kind of dynamic response cannot be managed through other means and the coupling of piers, mobilizing their ductility resources during the event can be beneficial. STUs become active for sudden actions such as railway braking loads, without however having any energy dissipation capacity in seismic conditions. The result is that STUs tend to stiffen the structure, lowering the natural period, thus negating the effects of the seismic isolation.

## 4 SEISMIC DEVICES FOR HSR BRIDGES

### 4.1 Isolation systems

Seismic isolation devices employed in HSR bridge applications include bearing isolation and energy dissipation systems.

Typical isolation systems employed are high-damping rubber bearings (HDRB), lead rubber bearings (LRB) and friction pendulum bearings (FPB). They have the multiple function of isolating the substructures and provide a degree of energy dissipation to the structural system. HDRB and LRB are elastomeric-based, displacement-dependent isolation systems, which exhibit elastoplastic with hardening behaviour. For HDRB, viscous damping may vary between 15-25%, while for LRB the range is 20-30%. FPB are sliding bearings having rigid-plastic and hardening behaviour, with effective viscous damping ratios that can vary between 5-25%.

## 4.2 Viscous dampers

Viscous dampers are energy dissipation systems employed to compensate and limit the displacements of the superstructure induced by the flexibility of the isolation system. They are suited to vibration control of HSR viaducts as they exhibit high levels of energy dissipation density, reducing displacements and forces transferred to the supports. The cycles force-displacement depend on the actuation speed through the relation

$$F = C \cdot v^\alpha \quad (1)$$

with  $F$  the response force,  $C$  and  $\alpha$  the damping coefficient and exponent respectively, which values control maximum force and nonlinear response of the device. The exponent  $\alpha$  usually varies between 0.1 and 0.3 in bridge applications. The smaller the value of  $\alpha$ , the faster the increase of the damping force at low velocities, the smaller the seismic force transmitted by the damper to the adjacent structures. Dampers with coefficients of up to 0.1, and lower, are available on the market.

### 4.2.1 Preliminary sizing of viscous dampers

The design of the substructures is carried out initially for non-seismic loads - gravity, co-existing traction and braking, deck linear variations - to meet the railway longitudinal limit displacements. The horizontal stiffness  $K_0$  required is the sum in parallel of the fixed supports stiffness of the bridge articulation. From the fundamental period of vibration  $T_0$ , entering the response spectrum, the corresponding spectral displacement  $d_e$  can be determined,

$$d_e = S_d(T_0) \quad (2)$$

Such a value is usually greater than the limit displacement imposed by the railway, meaning the substructures might respond in the plastic range in a seismic event without seismic devices. The seismic design criteria of the HSR specify the target limit displacement  $d_{cd}$ .

The simplification assumed at preliminary design stage is to model the viaduct as a pendulum comprising a mass, a spring stiffness and a nonlinear damper in series to the spring. Such a model allows the preliminary study of a regular straight viaduct with rigid deck.

For a reduction coefficient of the displacements equal to

$$\eta = \frac{d_{cd}}{d_e} \quad (3)$$

the global equivalent damping ratio is given by

$$\xi_{eq} = \frac{0.1}{\eta^2} - 0.05 \quad (4)$$

which represents the damping of an equivalent linear viscous damper that dissipates enough energy to reduce the displacements by a factor of  $\eta$ . The supplemental damping ratio provided by the viscous dampers is

$$\xi_D = \xi_{eq} - \xi_0 \quad (5)$$

with  $\xi_0$  the inherent damping ratio, set to 5% for reinforced concrete.

The damping coefficient is then

$$C = \frac{M}{h(\alpha)} \frac{4\pi}{T} \xi_D \left[ \frac{T}{2\pi} \eta S_e(T) \right]^{1-\alpha} \quad (6)$$

with  $M$  mass of the superstructure,  $T$  period of vibration of the undamped structure,  $S_e(T)$  the elastic response spectrum and  $h(\alpha)$  given by [Kahan, 2000]

$$h(\alpha) = \frac{2}{\pi} \int_0^\pi \sin^{\alpha+1} \vartheta d\vartheta \simeq 0.0892\alpha^2 - 0.3583\alpha + 1.2699 \quad (7)$$

The maximum force in the dampers is

$$F_{max} \simeq C \left[ \frac{T}{2\pi} \eta S_e(T) \right]^\alpha \quad (8)$$

and the dissipated energy

$$E_D \simeq 4F_{max}d_{cd} \quad (9)$$

for values of  $\alpha$  close to 0.1.

The damping ratio can then be expressed as

$$\xi_{eq} = \frac{E_D}{4\pi E_e} = \frac{2F_{max}}{\pi K_{eff}d_{cd}} \quad (10)$$

Viscous dampers are placed over the stiffer supports of the viaduct, usually at the shorter piers and abutments, where their action is more effective.

## 5 DISPLACEMENT-BASED DESIGN OF HSR BRIDGES: CONCEPT

Due to the performance requirements explicitly stated in terms of limit displacements of the superstructure, the seismic design of HSR bridges lend itself to be dealt within the framework of the displacement-based design (DBD) [Priestley et al, 2007].

In DBD, a multi-degree of freedom (MDOF) structure with nonlinear behaviour is replaced by a substitute structure [Shibata and Sozen, 1976] having a single degree of freedom (SDOF), which has the same secant stiffness at the design displacement, and with viscous damping increased due to the energy dissipation taking place during the seism. The target displacement represents the primary design quantity. Known the level of damping of the equivalent system corresponding to the target displacement, then the required period of vibration of the original structure can be determined from the appropriate displacement spectrum. Knowledge of the period of vibration allows to size the structure with the stiffness, strength and ductility required to meet the target displacement.

To meet the design requirement of elastic response of the substructures during the service earthquake, the design strategy is based on providing isolation between deck and substructure to reduce the seismic forces transferred to the stiffer supports and compensate the attendant increase in displacement with the provision of supplemental damping through viscous dampers.

With reference to a SDOF of the bridge and to the demand acceleration-displacement response spectrum (ADRS), Fig. 8, a series of elastic response spectra are shown, each associated to a damping level; straight lines passing through the origin of the axes are lines of iso-period. Both isolator-bearings and viscous dampers contribute, in different measure, to increase damping of the structural system. Due to their flexibility, increased damping through isolator-bearings is accompanied by an increase in displacements (direction A), compensated to the target displacement  $d_T$  by the provision of viscous dampers (direction B). The provision of isolators over the stiffer substructures softens the response, shifting the natural period of the system from  $T_f$  to  $T_{fis}$ , with  $T_{fis} > T_f$ . The period shifting is accompanied by the performance point moving to the damping curve corresponding to the range of the isolator employed. The difference between the damping level reached through the provision of the isolator and the one required to bring the system response within the target displacement, is the supplemental damping to be provided by the viscous dampers.

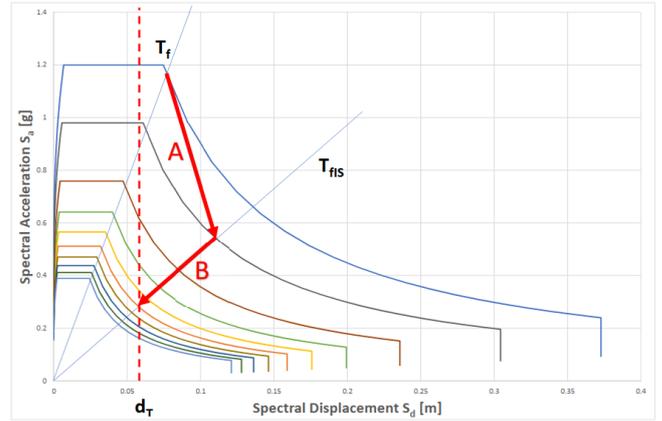


Figure 8. Design strategy in ADRS format

To determine the displacement response of the structure with viscous dampers, their effect on the effective damping of the substitute structure is taken into account, together with the hysteretic response of the isolators, while they do not affect the effective stiffness of the system. The effective stiffness of the system is equal to the sum of the contributions of all the isolation devices, assuming in the first instance their flexibility is much greater than that of the supporting piers. The deck seismic displacement is that of an equivalent linear SDOF system having the mass of the superstructure, effective stiffness  $K_{eff}$  and effective damping  $\xi_{eff}$  determined above. The peak displacement demand is derived from the displacement response spectrum multiplied by the damping modification factor  $\eta_{eff}$  corresponding to the estimated value of effective damping  $\xi_{eff}$ . The procedure is iterated to convergence, adjusting the design parameters of the structural system – the mechanical characteristics of the viscous dampers and isolators, the stiffness of the piers - until the target displacement is achieved.

As the method is based on the linearization of a nonlinear response through the modelling of a MDOF system with a SDOF model, sources of discrepancy with respect to the actual structural response may arise along the design process, more pronounced as less regular is the viaduct. At the detailed design stage, the verification of the target displacements through nonlinear time-history analysis should be carried out and update the design in accordance.

## 6 APPLICATION

To illustrate the design procedure, the type of viaducts designed for the Taiwan High-Speed Rail (THSR), completed in 2007, is considered. The

THSR runs North to South on the Western corridor of the island with a total length of 345 km, which 252 km are viaducts or bridges. The island is located on the Circum-Pacific Earthquake Belt and experiences several earthquakes annually.

The application considers a span configuration of 40 m with simply supported box-girder decks, the arrangement most used along the line, although continuous viaducts and special bridges have been built to cross demanding obstacles.

The seismic parameters employed are the type 1 horizontal elastic response spectrum according to [EN 1998-1], reference peak ground acceleration of 0.3g, ground type B modified with  $T_D = 2.5$  s. The limit displacement for the serviceability of the high-speed line is 25 mm in the longitudinal direction.

The structure comprises post-tensioned concrete box-girder of 4.00 m depth and reinforced concrete square hollow piers with wall thickness generally of 0.565 m and varying external sizes to suit the required horizontal stiffness to be provided under traction and braking, which is equal to 360 MN/N, for varying pier heights from 10 to 25 m, Fig. 10. Permanent load of 440 kN/m (250 kN/m deck dead load, 190 kN/m superimposed load).

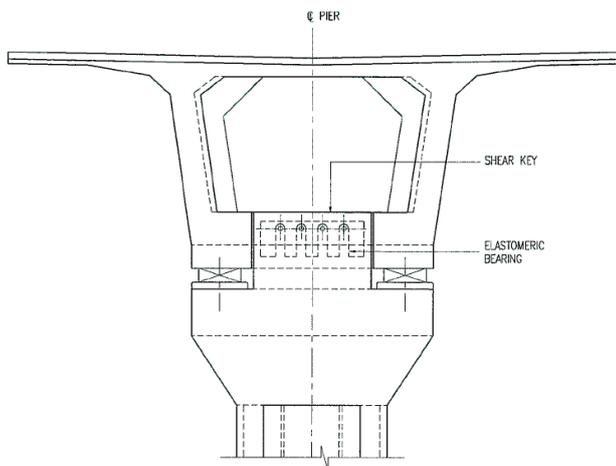


Figure 9. Deck outline of the application

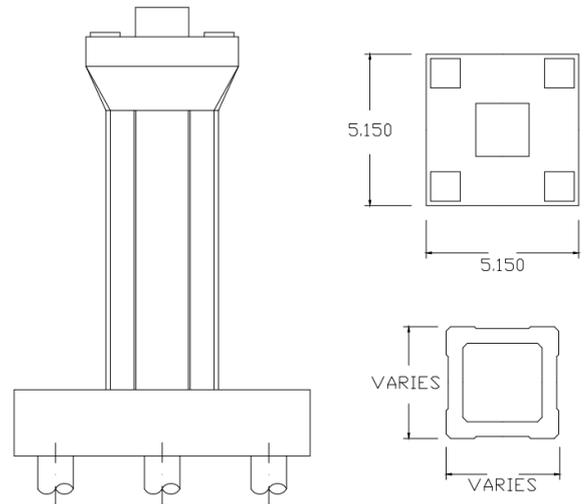


Figure 10. Substructure outline of the application

Fig. 11 displays the required supplemental damping in function of the height of the pier, the damping increasing with the height, that is with the flexibility of the substructures. The amount of damping to be provided in this instance is such that time-history non-linear analysis should be carried out at detailed design stage.

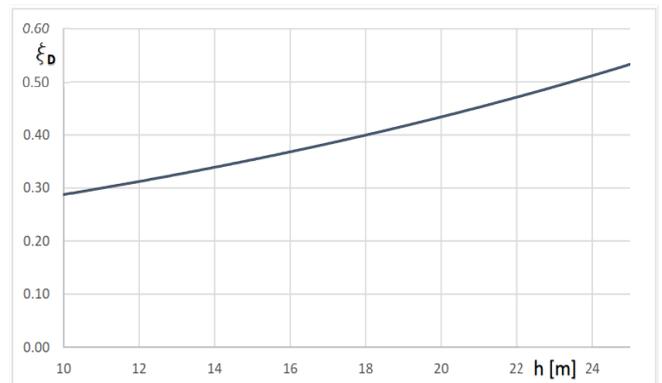


Figure 11. Supplemental damping with pier height

Fig. 12 shows the supplemental damping in function of the stiffness of the piers, here expressed as displacement at the top, varying between 3 and 5 mm, 5 mm representing the limit displacement under traction and braking. The graph has parameter the height of the piers, from 10 to 25 m with step increase of 2.5 m. It is seen that the shorter and stiffer is the pier, lesser is the damping demand required to ensure the maximum seismic displacement is within the stipulated limit, in accordance with Eq. (10).

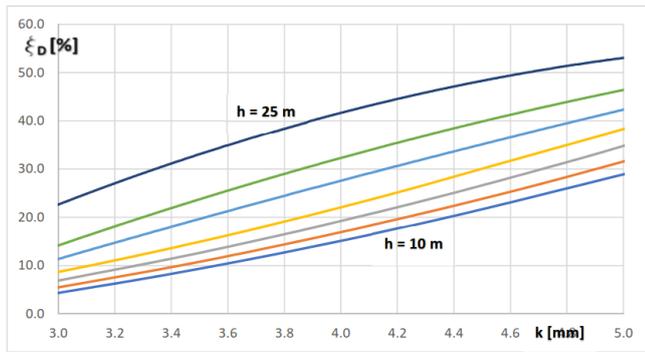


Figure 12. Supplemental damping with pier stiffness

Fig. 13 illustrates the maximum force in the dampers, normalized with respect to the corresponding longitudinal seismic force, the proportion of base shear filtered by the dampers, again with parameter the height of the pier, which confirms the increased demand as substructures become more flexible.

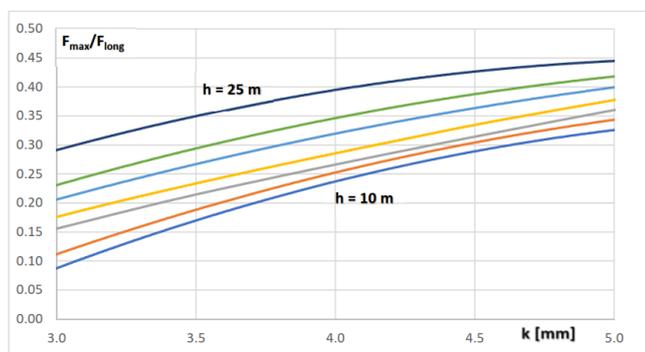


Figure 13. Maximum damper force with pier stiffness

## 7 CONCLUSIONS

The requirement to ensure the continuity of the functionality of the line after a frequent seismic event, has shifted the focus of the seismic design of high-speed rail viaducts and bridges towards the control of the displacements of the superstructure, rather than simply meeting strength and ductility criteria which would entail the structure responding locally in the plastic range, requiring repairs and closure of the line.

After having illustrated the principles of conceptual design of HSR viaducts for a range of configurations, a rational design procedure involving isolation and dissipation, suitable for preliminary seismic design, as well as for verification of more detailed analyses, has been presented, including an application to the viaducts employed in the Taiwan high-speed rail project.

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