# Optimal design of FPS for bridges in different soil conditions

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ABSTRACT
The work evaluates the optimal properties of friction pendulum system (FPS) devices for the seismic protection of bridge piers under earthquake excitations having different frequency characteristics representative of different soil conditions in order to reduce the seismic vulnerability of infrastructures. A two-degree-of-freedom model is adopted to describe, respectively, the response of the infinitely rigid deck isolated by the FPS devices and the elastic behavior of the pier. By means of a non-dimensional formulation of the motion equations, a wide parametric analysis for several structural parameters is carried out. Seismic excitations, modelled as time-modulated filtered Gaussian white noise random processes having different intensities and frequency contents, are considered. Specifically, the filter parameters, which control the frequency contents, are properly calibrated to reproduce stiff, medium and soft soil conditions, respectively. Finally, the optimum values of the sliding friction coefficient able to minimize the pier displacements with respect to the ground are derived as a function of the structural properties, of the seismic input intensity and of the soil condition.

## 1 INTRODUCTION

Seismic isolation of bridges makes it possible to uncouple the deck from the horizontal components of the earthquake motion, leading to a substantial reduction of the deck acceleration and, consequently, of the forces transmitted to the pier (Tsopelas et al. 1996), (Tongaonkar and Jangid 2003). In the last years, friction pendulum system (FPS) devices have often been preferred to other isolators for their capability of providing an isolation period independent of the mass of the supported structure, their capacity to assure high dissipation and recentering, and their longevity and durability properties (Ahmadi and Tadjbakhsh 1976-1992), (Landi et al. 2016). In (Castaldo and Tubaldi 2015), with reference to an equivalent two-degree-of-freedom (2dof) model for base-isolated building frames, a non-dimensionalization of the motion equation considering different isolator and system properties has been proposed. Contextually, other studies have been focused on the seismic response of bridge isolated with sliding pendulum isolators highlighting the advantages (Young-Suk and Chung-Bang 2007), (Murat and DesRoches 2008). Moreover, other works have been more oriented to develop design approaches for the isolators and to identify the optimal isolator properties. In this context, the seismic reliability-based design (SRBD) approach has been proposed and widely discussed in (Castaldo et al. 2015), (Palazzo et al. 2014) as a new methodology useful to provide design solutions for seismic devices taking into account the main uncertainties relevant to the problem. Jangid in (Jangid 2005), assuming a stochastic model of the earthquake ground motion, considered the seismic performance of a bridge equipped with FPS devices, characterized by a Coulomb behavior, illustrating that there exists an optimal value of the friction coefficient for which the pier base shear and deck acceleration can be minimized. Other works e.g. (Dicleli and Buddaram 2006), (Saritaş and Hasgür 2014), (Wai-Fah and Lian 2014), concerning isolated bridges have also demonstrated that soft soil condition leads to a higher demand in terms of displacements and shear forces by negatively influencing the isolated systems. In (Castaldo and

Tubaldi 2018), the optimal values of the friction coefficient taking into account the influence of the ground motion characteristics by means of the ratio between the Peak Ground Acceleration (PGA) and the Peak Ground Velocity (PGV) have been proposed.

This work investigates the influence of soil characteristics in terms of frequency content on the seismic performance of bridges isolated with FPS isolators to define the optimal sliding friction coefficients. The two-degree-of-freedom model, as employed in (Young-Suk and Chung-Bang 2007), (Masoud and Touraj 2012) is used for this purpose as an equivalent model representative of the dynamic behaviour of a single-column bent viaduct to describe, respectively, the seismic response of the infinitely rigid deck isolated by the FPS devices and of the elastic behavior of the pier. In compliance with the non-dimensionalization of the motion equations presented for base-isolated building frames in (Castaldo and Tubaldi 2015), in this study, a non-dimensionalization of the motion equations for isolated bridges is proposed in order to carry out a wide parametric analysis considering different values of the structural properties and three different sets of artificial ground motion records.

These latter ones are modelled as non-stationary stochastic processes and generated through the power spectral density method (Shinozuka and Deodatis 1991), with different frequency contents corresponding to stiff, medium and soft soil conditions (Pinto et al. 2004), respectively. Specifically, for each set of the random excitations, numerical simulations are executed to estimate the influence of the characteristic system and isolator properties on the response parameters relevant to the structural performance. Then, the optimal values of the sliding friction coefficient, able to minimize the pier displacements relative to the ground, are defined as a function of the structural parameters, of the seismic input intensity and of the soil condition.

## 2 NON-DIMENSIONAL MOTION EQUATIONS FOR ISOLATED BRIDGES

Assuming an equivalent 2dof model, the motion equations governing the response of a bridge equipped with single concave FPS devices (Figure ), subjected to the seismic input, $\ddot{u}_g(t)$ apply:

$$\begin{aligned} &m_d\ddot{u}_d(t)+m_d\ddot{u}_p(t)+c_d\dot{u}_d(t)+f_b(t)=-m_d\ddot{u}_g(t)\\ &m_p\ddot{u}_p(t)-c_d\dot{u}_d(t)+c_p\dot{u}_p(t)+\\ &\qquad +k_pu_p(t)-f_b(t)=-m_p\ddot{u}_g(t) \end{aligned} \qquad \text{(1a,b)}$$

where $u_d$ denotes the displacement of the deck relative to pier, $u_p$ the pier displacement relative to the ground, $m_d$ and $m_p$ respectively the mass of the deck and of the pier bridge, $k_p$ and $c_p$ respectively the pier stiffness and inherent viscous damping coefficient, $c_d$ the bearing viscous damping factor, $t$ the time instant, the dot differentiation over time, and $f_b(t)$ indicates the FPS force, that can be evaluated as:

$$f_b(t)=k_du_d(t)+\mu(\dot{u}_d)m_dg\,\mathrm{sgn}(\dot{u}_d) \qquad (2)$$

where $k_d=W/R=m_dg/R$, $g$ is the gravity constant, $R$ is the radius of curvature of the FPS, $\mu(\dot{u}_d(t))$ the sliding friction coefficient, which depends on the bearing slip velocity $\dot{u}_d(t)$, and $sgn(\cdot)$ denotes the sign function. It follows that, similarly to base-isolated structures (Castaldo and Tubaldi 2015), the fundamental vibration period of an isolated bridge, $T_d=2\pi\sqrt{R/g}$, corresponding to the pendulum component, is independent of the deck mass and related only to the radius of curvature *R*.

According to (Mokha et al. 1990), (Constantinou et al. 2007), the sliding friction coefficient of teflon-steel interfaces can be expressed as:

$$\mu(\dot{u}_d)=f_{\max}-Df\cdot\exp(-\alpha|\dot{u}_d|) \qquad (3)$$

where $f_{\max}$ and $f_{\min}=f_{\max}-Df$ represent, respectively, the maximum value of sliding friction coefficient attained at large velocities and the value at zero velocity. In this study, it is considered that $f_{\max}=3f_{\min}$ with the exponent $\alpha$ equal to 30 (Castaldo and Tubaldi 2015).

Considering the maximum value of the sliding friction coefficient, the effective stiffness of the FPS bearings $k_{eff}=W(1/R+f_{\max}/u_d)$ as well as the corresponding effective isolated period $T_{d,eff}$ (Kelly 1997), (Building Seismic Safety Council 2006), (Figure 1) can be computed depending on the displacement demand. Note that Equations (1a,b) does not consider the effects of the higher modes due to flexibility of the pier and is verified if only the horizontal component of the bearing displacement is considered (Castaldo et al. 2017) (i.e., high radii of curvature *R*). Furthermore, the equivalent 2dof model (Young-Suk and Chung-Bang 2007), (Masoud and Touraj 2012) can be assumed representative of the dynamic behaviour of a single-column bent viaduct as long as the bridge is straight and consists of a large number of equal spans, of piers with equal height/stiffness and considering a superstructure (deck) that can be assumed to move as a rigid body (Priestley et al. 1996).

Let us introduce the time scale $\tau = t\omega_d$ in which $\omega_d = \sqrt{k_d / m_d}$ is the fundamental circular frequency of the isolated system with infinitely rigid superstructure, and the seismic intensity scale $a_0$, expressed as $\ddot{u}_g(t) = a_0 \ell(\tau)$ where $\ell(\tau)$ is a non-dimensional function of time describing the seismic input time-history, the following non-dimensional equations can be obtained and herein proposed for isolated bridges:

$$\begin{aligned} &\ddot{\psi}_d(\tau) + \ddot{\psi}_p(\tau) + 2\xi_d \dot{\psi}_d(\tau) + \\ &+\psi_d(\tau) = -\ell(\tau) - \frac{\mu(\dot{u}_d)\, g}{a_0}\operatorname{sgn}(\dot{\psi}_d) \\ &\ddot{\psi}_p(\tau) - \frac{1}{\lambda}\left[2\xi_d \dot{\psi}_d(\tau) + \psi_d(\tau) + \frac{\mu(\dot{u}_d)\, g}{a_0}\operatorname{sgn}(\dot{\psi}_d)\right] + \\ &+2\xi_p \frac{\omega_p}{\omega_d}\dot{\psi}_p(\tau) - \frac{\omega_p^2}{\omega_d^2}\psi_p(\tau) = -\ell(\tau) \end{aligned} \qquad \text{(4a,b)}$$

where $\omega_p = \sqrt{k_p / m_p}$ and $\xi_p = c_p / 2 m_p \omega_p$ represent respectively the circular frequency and damping factor of the pier bridge; $\omega_d = \sqrt{k_d / m_d} = \sqrt{g / R}$ and $\xi_d = c_d / 2 m_d \omega_d$ are respectively the circular frequency and the isolator damping factor of the FPS isolator; $\lambda = m_p / m_d$ (Young-Suk and Chung-Bang 2007), (Masoud and Touraj 2012), (Kelly 1997) the mass ratio. The non-dimensional parameters $\psi_d = \frac{u_d \omega_d^2}{a_0}$ and $\psi_p = \frac{u_p \omega_d^2}{a_0}$ describe the dynamic response of the deck and the pier, respectively.

From Equations (4a,b), it is possible to observe that the five non-dimensional Π terms (Castaldo and Tubaldi 2015), (Karavasilis et al. 2011), (Barbato and Tubaldi 2013) that govern the system non-dimensional response are:

$$\Pi_\omega = \frac{\omega_p}{\omega_d},\ \Pi_\lambda = \lambda,\ \Pi_\mu(\dot{\psi}_d) = \frac{\mu(\dot{u}_d)\, g}{a_0},\ \Pi_{\xi_d} = \xi_d,\ \Pi_{\xi_p} = \xi_p \qquad \text{(1a,b,c,d,e)}$$

where $\Pi_\omega$ represents the isolation degree (Kelly 1997), (Palazzo 1991), $\Pi_\lambda$ is the mass ratio as previously defined, $\Pi_{\xi_p}$ and $\Pi_{\xi_d}$ are related to the inherent viscous damping of the pier and the isolator, respectively, $\Pi_\mu$ denotes the isolator strength which depends on both the friction coefficient $\mu(\dot{u}_d)$ and the seismic intensity. Since the sliding friction coefficient is a velocity-dependent parameter, $\Pi_\mu$ is considered as follows (Castaldo and Tubaldi 2015):

$$\Pi_\mu^* = \frac{f_{\max}\, g}{a_0} \qquad (6)$$

From Equations (4a,b)-(6), note that only the non-dimensional terms $\Pi_{\xi_d}$, $\Pi_{\xi_p}$, $\Pi_\omega$, $\Pi_\lambda$, $\Pi_\mu^*$, the function $\ell(\tau)$, describing the frequency content and time-modulation of the seismic input, and the time scale parameter $\omega_d$ influence the non-dimensional seismic response of the bridge system isolated by FPS.

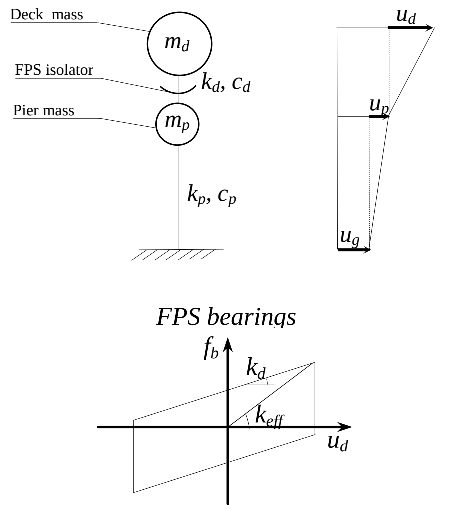



Figure 1. 2dof model of a bridge isolated by FPS bearings.

## 3 UNCERTAINTIES RELATED TO THE SEISMIC INPUT

This section describes the stochastic model employed for the generation of the artificial ground motions in order to reproduce the uncertainty in terms of frequency characteristics for different soil conditions as well as the uncertainty corresponding to the seismic intensity.

### 3.1 Random excitations

The "record-to-record" variability in terms of the dynamic characteristics of different seismic inputs related to stiff, medium and soft soil conditions, respectively, is herein described by means of three corresponding wide groups of artificial records having different frequency contents. These artificial inputs are modelled as time-modulated filtered Gaussian white noise random processes (Shinozuka and Deodatis 1991), (Pradlwarter et al. 1998) within the power spectral density (PSD) method (Tung et al. 1992) by adopting the Kanai-Tajimi model (Kanai 1957), (Tajimi 1960), modified by Clough and Penzien (Clough and Penzien 1993), (Saritaş and Hasgür 2014), (Zentner et al. 2014), (Tubaldi et al. 2012), as follows:

$$S_f(\omega) = \frac{\omega_g^4 + 4\xi_g^2\omega_g^2\omega^2}{(\omega_g^2-\omega^2)+4\xi_g^2\omega_g^2\omega^2} \cdot \\ \cdot \frac{\omega^4}{(\omega_f^2-\omega^2)+4\xi_f^2\omega_f^2\omega^2} S_0 \qquad (7)$$

in which $S_0$ is the amplitude of the bedrock excitation spectrum, modeled as a white noise process; $\omega_f$ and $\xi_f$ are the Clough-Penzien filter parameters assumed as deterministic values, set equal to $\omega_f = 1.6$ (rad/s) and $\xi_f = 0.6$; $\omega$ is the circular frequency, assumed varying in the range 0 and 50 rad/s; $\omega_g$ and $\xi_g$ represent the fundamental circular frequency and damping factor of the soil, respectively, assumed as uniformly distributed independent random variables with appropriate ranges of variation (Pinto et al. 2014), (Talaslidis et al. 2004) as follows: $\omega_g$ varies in the range 5π-9π rad/s (high frequency/short period) with $\xi_g$ =0.6-1 for stiff soil condition, $\omega_g$ is assumed ranging between 3π rad/s and 5π rad/s (intermediate frequency/ period) with $\xi_g$ =0.4-0.6 for medium soil condition, and, finally, $\omega_g$ ranges from π to 3π (low frequency/high period) with $\xi_g$ =0.2-0.4 for soft soil condition. Specific sampling techniques (Castaldo et al. 2018a), (Castaldo et al. 2018b) are used to sample the data. Assuming the same duration (Hancock and Bommer 2006), (Hancock and Bommer 2007) equal to 31.25 s, longer than 25s as provided from (NTC2008 2008), the Shinozuka-Sato function (Shinozuka and Sato 1967) is adopted as time-modulating function in order to define non-stationary stochastic processes for each set corresponding to each soil condition. Specifically, 100 artificial (non-stationary stochastic processes) records, generated through the Spectral Representation Method (Shinozuka and Deodatis 1991) and reflecting the wide uncertainty in terms of frequency content for each soil type (Pinto et al. 2004), (Talaslidis et al. 2004), (Armouti 2003) are defined for each soil condition.

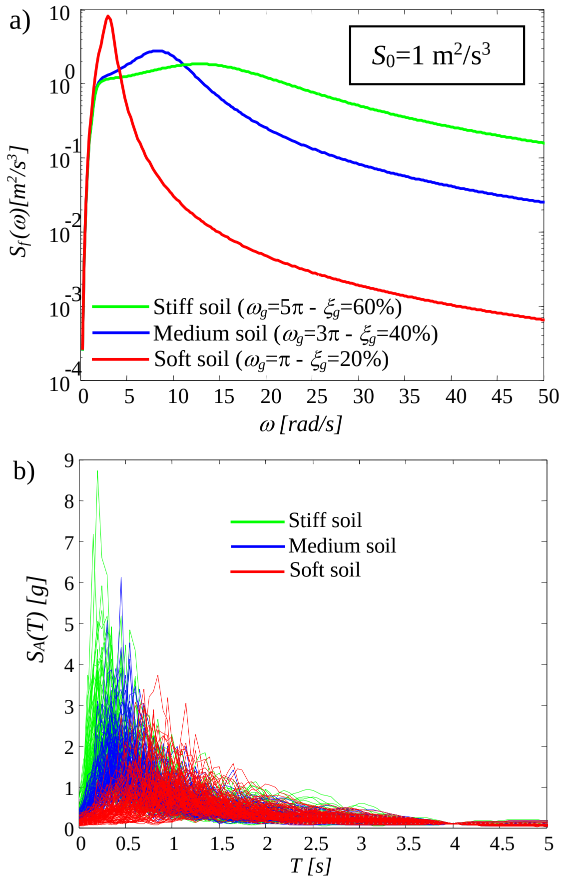



Figure 1 . PSD functions corresponding to stiff, medium and soft soil conditions (a); Pseudo-acceleration response spectra for the 300 records scaled to the common seismic intensity measure $S_A(T)$ = 0.1 g, for $T$=4s (b).

Note also that, for each set of artificial records a high number of random excitations is defined in order to highly reduce the standard errors of the statistics of the response parameters (Castaldo et al. 2017). As an example, Figures 2(a)-(b) show, respectively, the sampled PSD functions and the elastic pseudo-acceleration response spectra of the 300 artificial records, scaled to the common *IM* value $S_A(T)$ = 0.1g, for a period $T$ = 4s.

### 3.2 Intensity measure

In order to take into account the uncertainty related to the seismic intensity, the intensity scale factor, $a_0$, of Equations (
**Errore. L'origine riferimento non è stata trovata.** a,b), represents the seismic intensity measure (*IM)* in the context of the performance-based earthquake engineering (PBEE) (Aslani and Miranda 2005), (Porter 2003). In this study, the abovementioned *IM* is denoted by the spectral pseudo-acceleration, $S_A\left(T_d, \xi_d\right)$, corresponding to the isolated period of the bridge $T_d = 2\pi / \omega_d$ with the damping ratio $\Pi_{\xi_d} = \xi_d$. Note that, in the analyses herein developed, the damping ratio $\xi_d$ is set equal to zero (Castaldo and Tubaldi 2015), (Jangid 2005), (Ryan and Chopra 2004) and the corresponding *IM* is hereinafter denoted as $S_A\left(T_d\right)$.

## 4 PARAMETRIC STUDY

This section describes the results of the parametric study carried out on the system of Figure to evaluate the seismic performance of bridge isolated with FPS bearings for different structural properties and soil conditions. The first subsection describes the response parameters relevant to the seismic performance, whereas the final subsection illustrates the parametric study results. More details may be found in (Castaldo et al. 2018c).

### 4.1 Non-dimensional response parameters relevant to the seismic performance assessment

The following response parameters relevant to the seismic performance assessment of isolated bridges are considered: the peak deck displacement relative to the pier $u_{\mathrm{d,max}}$, the peak pier displacement $u_{\mathrm{p,max}}$. These latter ones can be defined in non-dimensional form, as expressed in Equations (4a,b), as:

$$\psi_{u_d} = \frac{u_{\mathrm{d,max}}\omega_d^2}{S_A\left(T_d\right)} = \frac{u_{\mathrm{d,max}}}{S_d\left(T_d\right)}, \qquad \psi_{u_p} = \frac{u_{\mathrm{p,max}}\omega_d^2}{S_A\left(T_d\right)} = \frac{u_{\mathrm{p,max}}}{S_d\left(T_d\right)} \tag{8a,b}$$

For each soil condition (i.e., set of the 100 ground motion records), Equations (4a,b) is repeatedly solved computing a set of samples for each response parameter. As also described in (Castaldo and Tubaldi 2015), (Palazzo et al. 2014), (Ryan and Chopra 2004), (Karavasilis and Seo 2011), the response parameters are modeled in probabilistic terms by means of a lognormal distribution. Specifically, the generic response parameter *D* (i.e., the extreme values $\psi_{u_d}$, $\psi_{u_p}$ of Equation (4a,b)) can be fitted by a lognormal distribution estimating the sample geometric mean, $GM\left(D\right)$, and the sample lognormal standard deviation $\sigma_{\ln}\left(D\right)$, or dispersion $\beta\left(D\right)$, defined, respectively:

$$GM\left(D\right) = \sqrt[N]{d_1 \cdot \ldots \cdot d_N} \tag{9}$$

$$\beta\left(D\right) = \sigma_{\ln}\left(D\right) = = \sqrt{\frac{\left(\ln d_1 - \ln\left[GM\left(D\right)\right]\right)^2 + .. + \left(\ln d_N - \ln\left[GM\left(D\right)\right]\right)^2}{N-1}} \tag{10}$$

in which $d_i$ is the *i*-th sample value of *D*, and *N* represents the total number of samples. The *k*th percentile of the generic response parameter *D* can be evaluated as:

$$d_k = GM(D)\exp[f(k)\beta(D)] \tag{11}$$

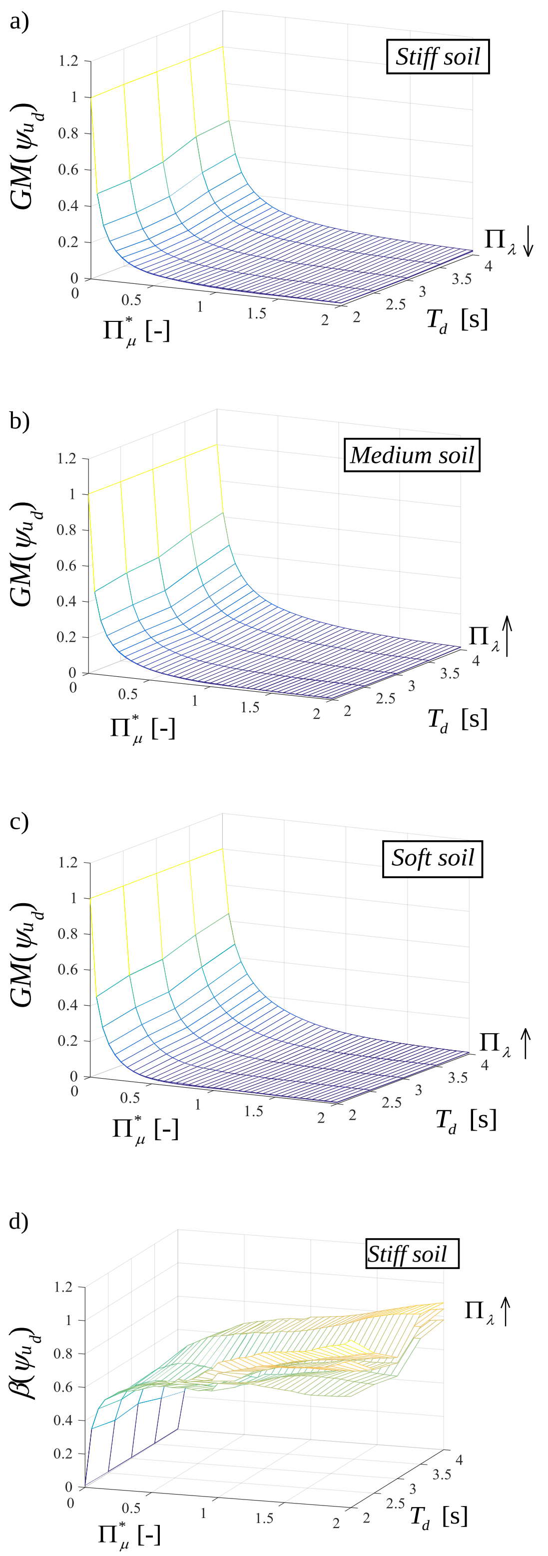
where $f(k)$ is a function that assumes the following values $f(50)=0$, $f(84)=1$ and $f(16)=-1$ (Ang and Tang 2007), for the 50th, 16th and 84th percentile, respectively.

### 4.2 Parametric study results for each soil condition

In this section, the results of the parametric study developed using the proposed non-dimensionalization, for the different structural properties and for each set of 100 records, are illustrated and discussed. According to several studies (Tsopelas at al. 1996), (Jangid 2004), (Tongaonkar and Jangid 2003), (Young-Suk and Chung-Bang 2007), (Murat and DesRoches 2008), (Masoud and Touraj 2012), (Yen-Po et al. 1998), (Jangid 2008), the parameters $\Pi_{\xi_d} = \xi_d$ and $\Pi_{\xi_p} = \xi_p$ are assumed respectively equal to 0% and 5%, the isolation period $T_d$ varies in the range between 2s and 4s, the pier period $T_p$ ranges from 0.05s to 0.2s, $\Pi_\lambda = \lambda$ varies between

0.1 and 0.2, $\Pi^*_\mu$ ranges between 0 (no friction) and 2 (very high friction) (Castaldo and Tubaldi 2015). Other uncertainties are not considered. Indeed, a high value for the upper bound of $\Pi^*_\mu$ is considered in order to take also into account the very low values of the *IM* at high isolated periods (i.e., $T_d$=4s) depending on the seismic hazard (NTC2008 2008). For each parameter combination, the differential motion equations, i.e., Equations (4a,b), have been repeatedly solved adopting the Bogacki-Shampine integration algorithm available in Matlab-Simulink (Math Works Inc. 1997). After that, for each normalized response parameter, the geometric mean, *GM*, and the dispersion, $\beta$, have been evaluated through Equations (9) and (10) and are plotted in Figures. 3-10 for each soil type. Each figure contains several meshes, corresponding to the different $\Pi_\lambda$. The results for deck and pier displacements related to the all pier periods are reported.

Figures 5-8 plot the results concerning the normalized deck displacement $\psi_{u_d}$, related to different pier period values. $GM\left(\psi_{u_d}\right)$ is quite perfectly equal to unit for $\Pi^*_\mu = 0$ and $T_p = 0.05$ because of the very reduced influence of the pier behaviour. For $\Pi^*_\mu \neq 0$, and $GM\left(\psi_{u_d}\right)$ increases slightly for increasing $T_d$ because of the period elongation. Obviously, $GM\left(\psi_{u_d}\right)$ decreases significantly as $\Pi^*_\mu$ increases while it is not heavily influenced by $\Pi_\lambda$. For soft soil condition and low $\Pi^*_\mu$ values, the decrease of $GM\left(\psi_{u_d}\right)$ for increasing $\Pi^*_\mu$ is more gradual, while, for high $\Pi^*_\mu$ values $GM\left(\psi_{u_d}\right)$ increases in the case of stiff soil, especially, for high $T_p$ values due to the pier influence. The dispersion $\beta\left(\psi_{u_d}\right)$ for high $T_d$ increases for increasing values of $\Pi^*_\mu$, as a result of the reduction of the efficiency of the *IM* employed in the study for each soil condition. Moreover, with reference to soft soils, the values of $\beta\left(\psi_{u_d}\right)$ also result to be the highest for low or high values of both $T_d$ and $\Pi^*_\mu$. Obviously, in the reference situation corresponding to $\Pi^*_\mu = 0$ and $T_p$=0.05s, the dispersion is zero for all the values of $T_d$ and of $\Pi_\lambda$ considered and for all the soil conditions. The mass ratio $\Pi_\lambda$ does not affect significantly the response dispersion, especially in the case of high $T_p$ values.



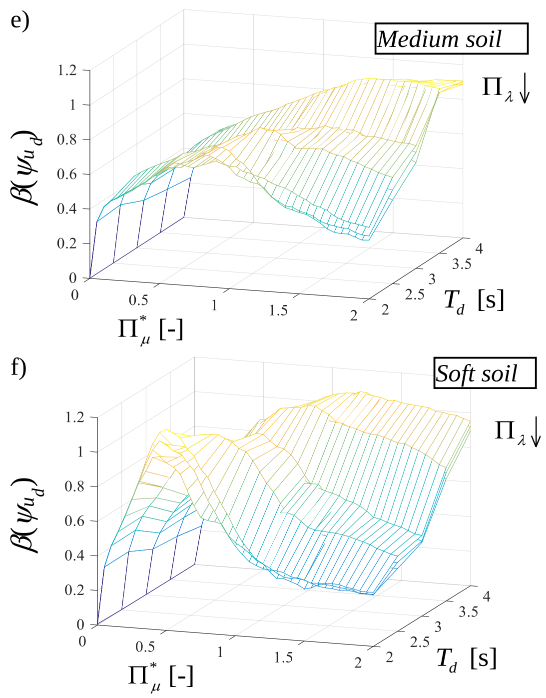
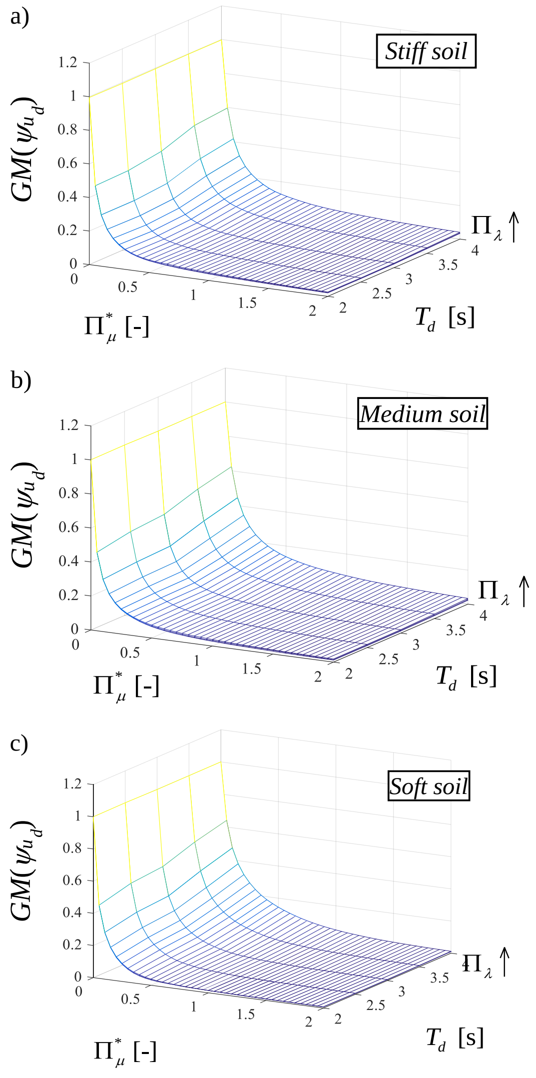



Figure 3 . Normalized deck displacement vs. $\Pi^*_\mu$ and $T_d$ for $T_p$ =0.05s and each soil condition: median value (a,b,c) and dispersion (d,e,f) for different values of $\Pi_\lambda$. The arrow denotes the increasing direction of $\Pi_\lambda$.



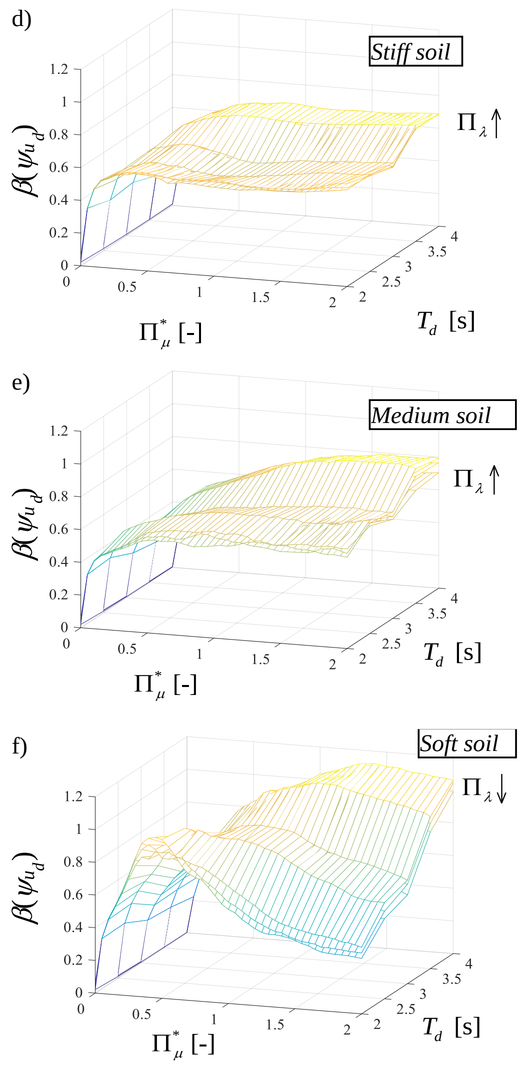
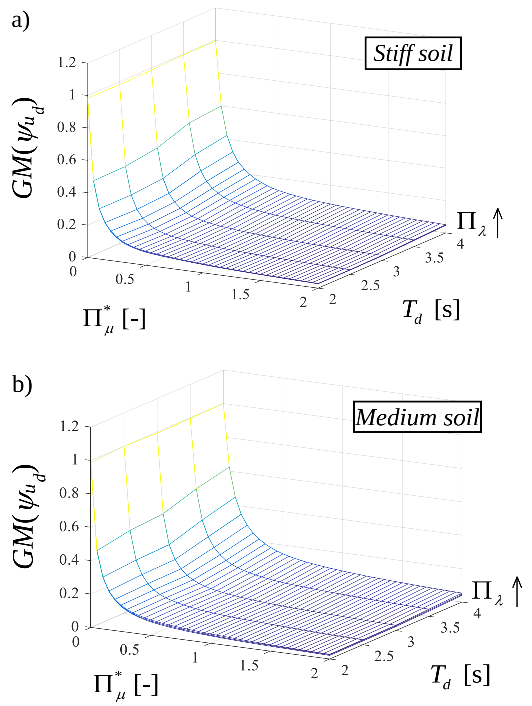




Figure 4 . Normalized deck displacement vs. $\Pi^*_\mu$ and $T_d$ for $T_p$ =0.1s and each soil type: median value (a,b,c) and dispersion (d,e,f) for different values of $\Pi_\lambda$. The arrow denotes the increasing direction of $\Pi_\lambda$.



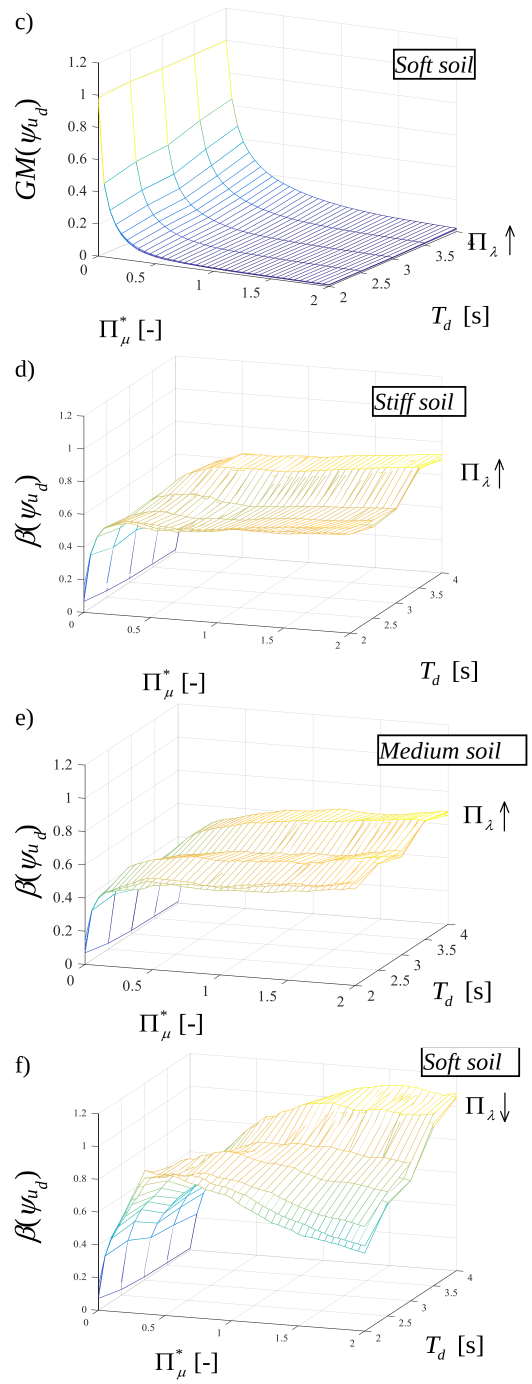
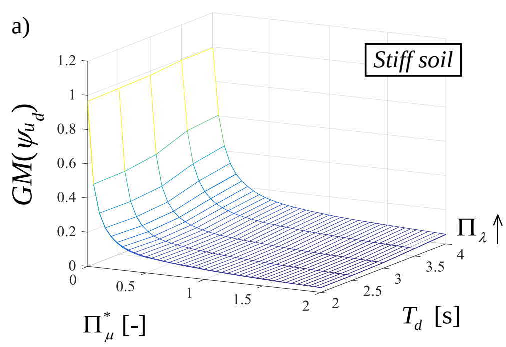



Figure 5 . Normalized deck displacement vs. $\Pi^*_\mu$ and $T_d$ for $T_p$ =0.15s and each soil type: median value (a,b,c) and dispersion (d,e,f) for different values of $\Pi_\lambda$. The arrow denotes the increasing direction of $\Pi_\lambda$.

The above described peak values of both $GM\left(\psi_{u_d}\right)$ and $\beta\left(\psi_{u_d}\right)$ in the case of soft soil condition are high due to the resonance effects which mainly affect the effective frequency charactering the dynamic behaviour of the frictional bearings and the dominant frequency of the corresponding random excitations.

Figures. 6-10 show the response statistics of the normalized pier displacements $\psi_{u_p}$ . $GM\left(\psi_{u_p}\right)$ decreases for increasing values of $T_d$ and of $\Pi_\lambda$ as well as for decreasing values of $T_p$, whereas it first decreases and then increases for increasing values of $\Pi^*_\mu$. Thus, this means that there exists an optimal value of the normalized friction coefficient $\Pi^*_\mu$ such that the pier displacement is minimized for each soil condition. This optimal value is in the range between 0.1 and 0.3 and depends on the values of $T_p$, $T_d$, $\Pi_\lambda$ and on the soil condition. Differently to the case of base-isolated systems, there is not a particular and specific trend of the optimal friction coefficients from stiff to soft soil condition, as discussed later in detail. There is a further increase in the value of $GM\left(\psi_{u_p}\right)$ from soft soil to stiff soil due to resonance effects, especially, for lower values of $T_d$. The values of the dispersion $\beta\left(\psi_{u_p}\right)$ are very low for low $\Pi^*_\mu$ values due to the high efficiency of the *IM* used in this work, and attain their peak for values of $\Pi^*_\mu$ close to the optimal ones. The other system parameters have a reduced influence on $\beta\left(\psi_{u_p}\right)$ compared to the influence of $\Pi^*_\mu$. For the soft soil condition, the dispersion $\beta\left(\psi_{u_p}\right)$ strongly increases for increasing values of $\Pi^*_\mu$ for low isolation period and for higher pier periods because of the resonance effects which mainly affect the effective frequency of the frictional bearings and the dominant frequency of the corresponding random excitations.



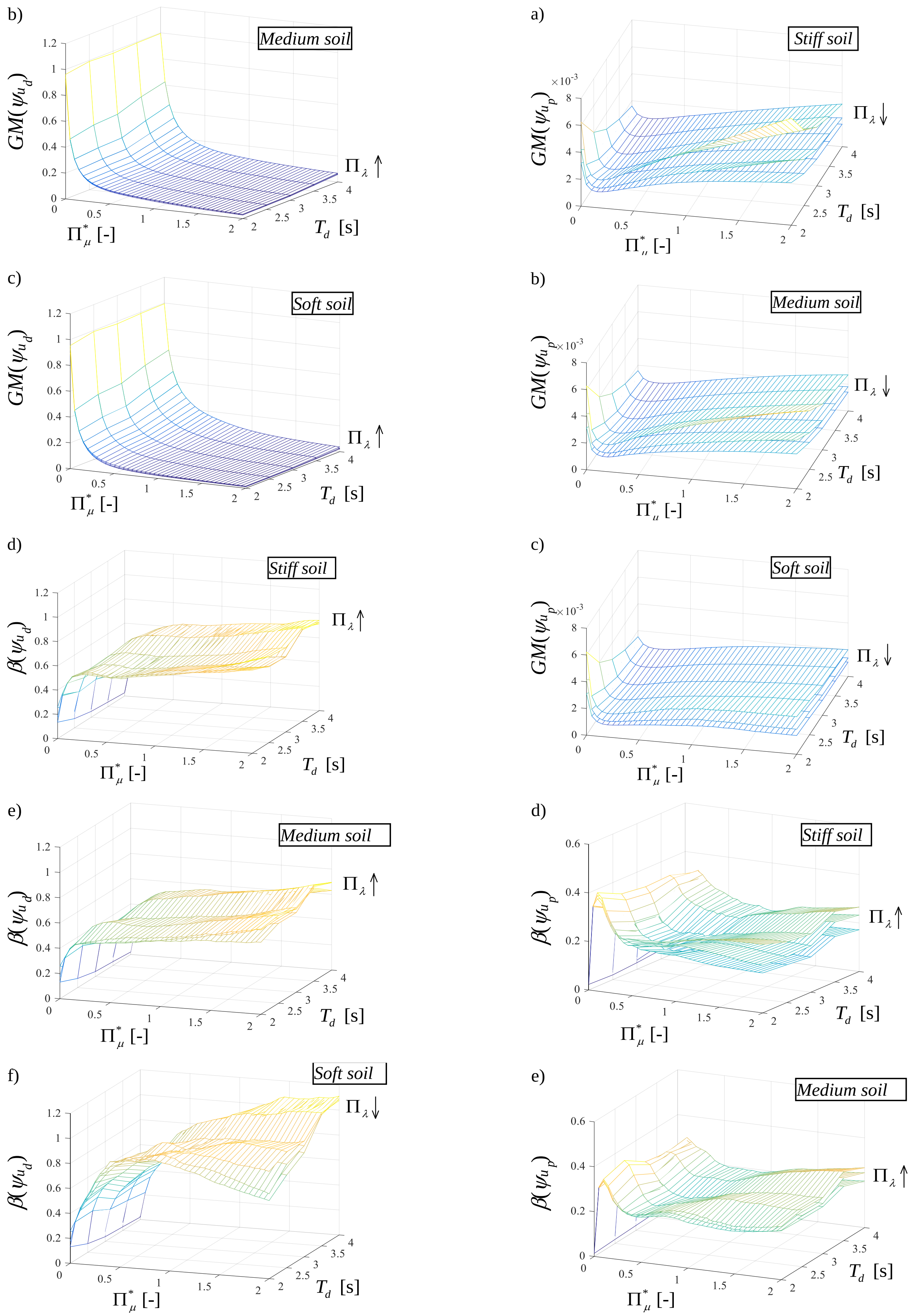



Figure 6 . Normalized deck displacement vs. $\Pi^*_\mu$ and $T_d$ for $T_p$ =0.2s and each soil type: median value (a,b,c) and dispersion (d,e,f) for different values of $\Pi_\lambda$. The arrow denotes the increasing direction of $\Pi_\lambda$.

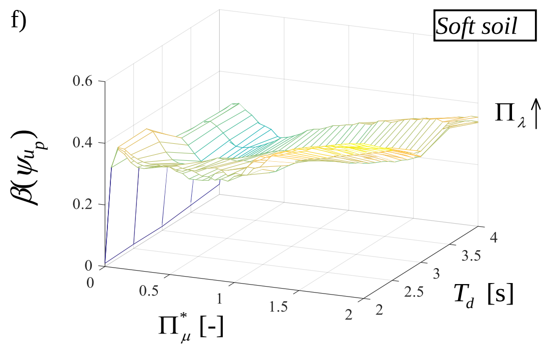
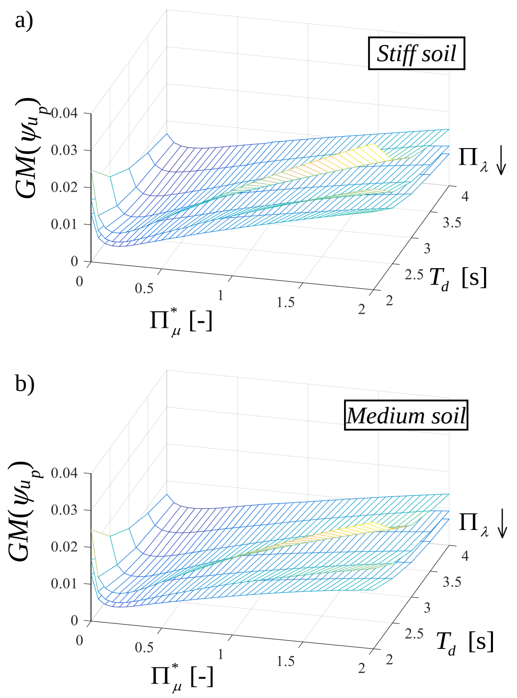



Figure 7. Normalized pier displacement vs. $\Pi^*_\mu$ and $T_d$ for $T_p$ =0.05s and each soil condition: median value (a,b,c) and dispersion (d,e,f) for different values of $\Pi_\lambda$. The arrow denotes the increasing direction of $\Pi_\lambda$.

As observed in similar studies (Castaldo and Tubaldi 2015), (Jangid 2005), (Chung et al. 2013), (Fallah and Zamiri 2013), the existence of an optimal value of the friction coefficient derives from a combination of different effects. Indeed, an increase of the sliding friction coefficient leads to higher isolator strengths (and thus higher values of the equivalent stiffness, with a lowering of the corresponding effective fundamental vibration period ) and higher forces towards the deck. This also leads to an increase in the forces transmitted to the pier bridge due to inertial effect, relative to deck mass, on the pier.



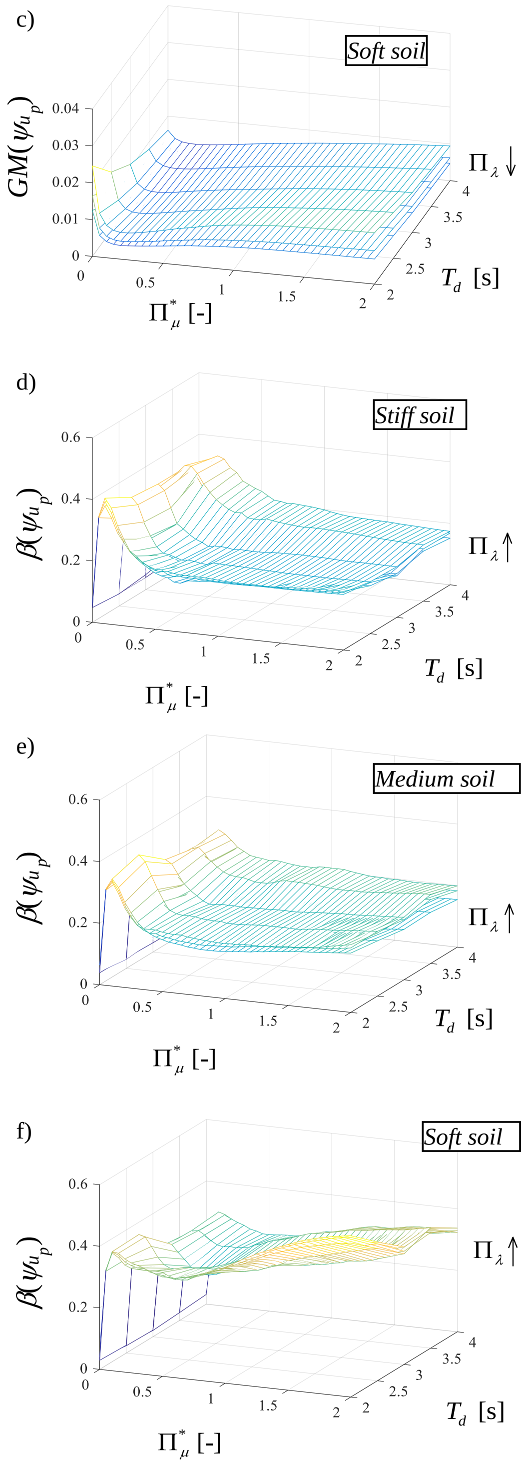
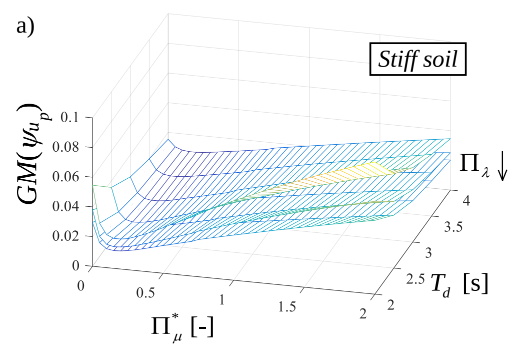




Figure 8. Normalized pier displacement vs. $\Pi^*_\mu$ and $T_d$ for $T_p$ =0.1s and each soil condition: median value (a,b,c) and dispersion (d,e,f) for different values of $\Pi_\lambda$. The arrow denotes the increasing direction of $\Pi_\lambda$.



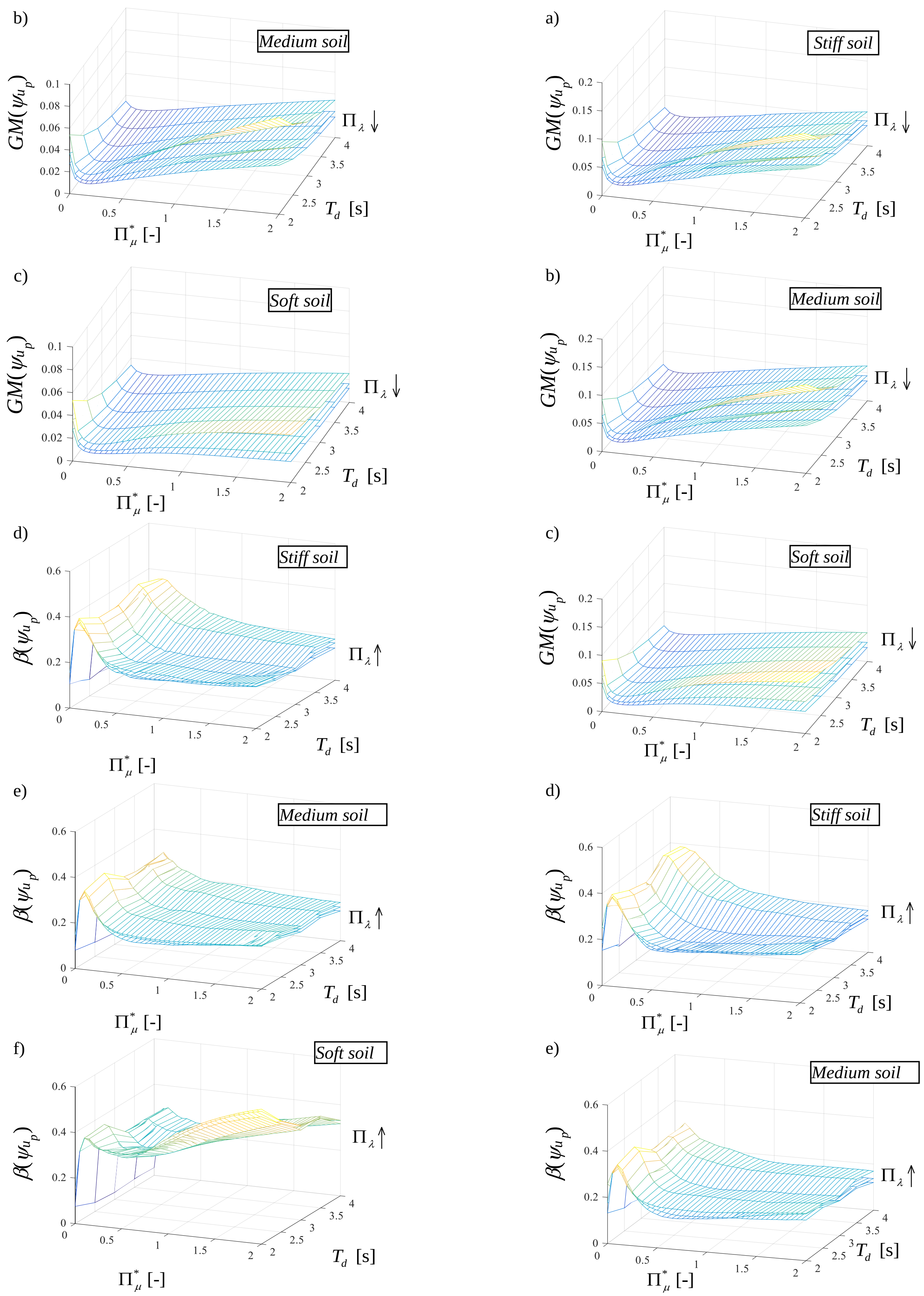



Figure 9 . Normalized pier displacement vs. $\Pi^*_\mu$ and $T_d$ for $T_p$ =0.15s and each soil condition: median value (a,b,c) and dispersion (d,e,f) for different values of $\Pi_\lambda$. The arrow denotes the increasing direction of $\Pi_\lambda$.

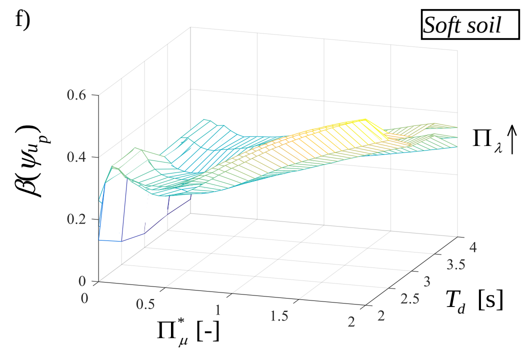
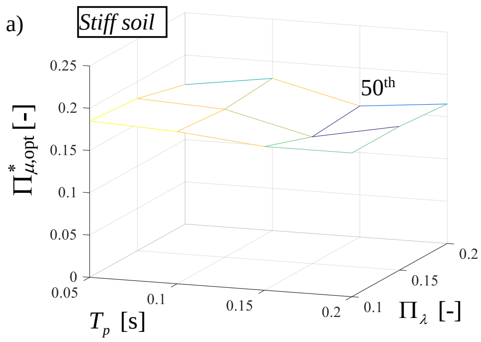



Figure 10 . Normalized pier displacement vs. $\Pi^*_\mu$ and $T_d$ for $T_p$ =0.2s and each soil condition: median value (a,b,c) and dispersion (d,e,f) for different values of $\Pi_\lambda$. The arrow denotes the increasing direction of $\Pi_\lambda$.

# 5 OPTIMAL SLIDING FRICTION COEFFICIENTS FOR ISOLATED BRIDGES DEPENDING ON SOIL CONDITIONS

From the results defined in the previous section, for each parameter combination (i.e., $\Pi_\lambda$, $T_d$ and $T_p$) and soil condition, the optimal values of the normalized sliding friction coefficient, $\Pi^*_{\mu,\mathrm{opt}}$, that minimize the median (50$^{th}$ percentile) normalized pier displacements $\psi_{u_p}$ have been computed and are illustrated in Figure 11. Minimizing the pier displacements relative to the ground represents a notable design requisite for the safety of bridges in order to assure an adequate seismic protection. In fact, an inelastic response of the pier can lead to a disproportionately large displacement response that could also be amplified in the case of the resonance effects. Figure 11 shows the variation of $\Pi^*_{\mu,\mathrm{opt}}$ with $\Pi_\lambda$ and $T_p$ for $T_d$ = 2s (Figure 11a,b,c) and $T_d$ = 4s (Figure 11d,e,f), for the three soil conditions.



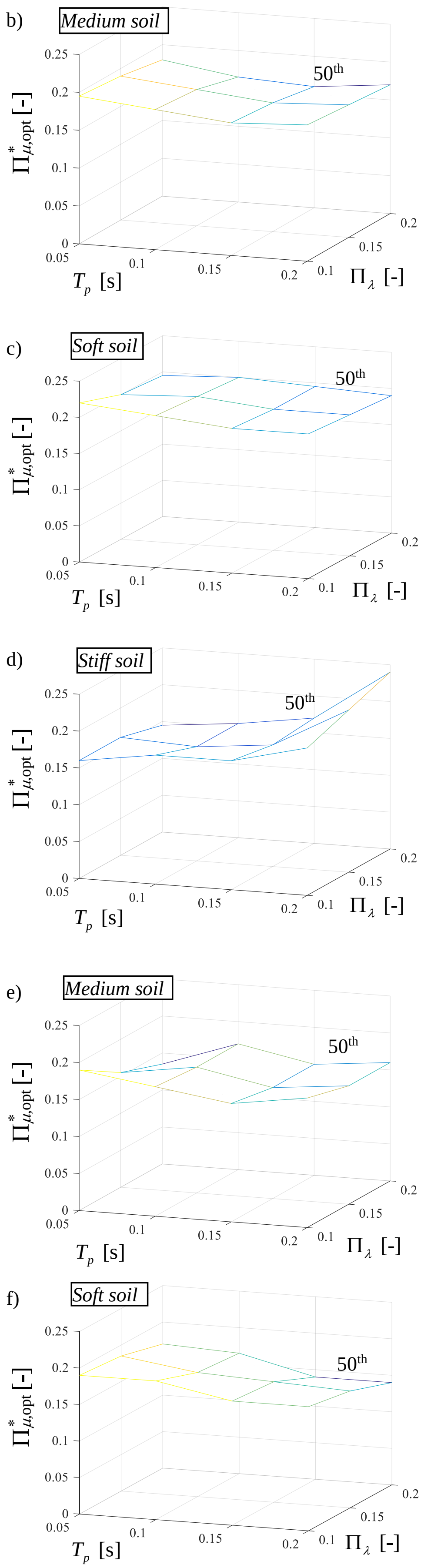
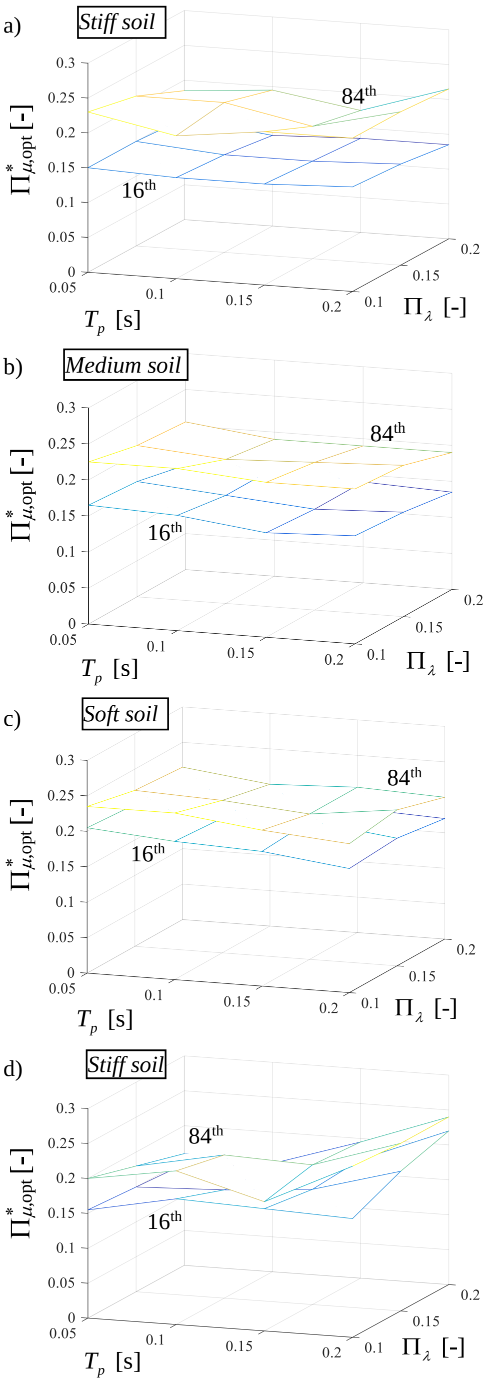




Figure 11 . Optimal values of normalized friction that minimize the 50$^{th}$ percentile of the normalized pier displacements vs. $\Pi_\lambda$ and $T_p$ for each soil type and for $T_d$ =2s (a,b,c) and $T_d$ =4s (d,e,f).

According to (Jangid 2005), the optimal values of the (normalized) sliding friction coefficient slightly increase for decreasing $T_d$, especially for low $T_p$ and for each soil condition. It is also observed that, for low $T_d$, $\Pi^*_{\mu,\mathrm{opt}}$ generally decreases by increasing $\Pi_\lambda$ and $T_p$. This trend is reversed with increasing of $T_d$ and soil stiffness, when it is necessary to dissipate more energy, due to the resonance effects.



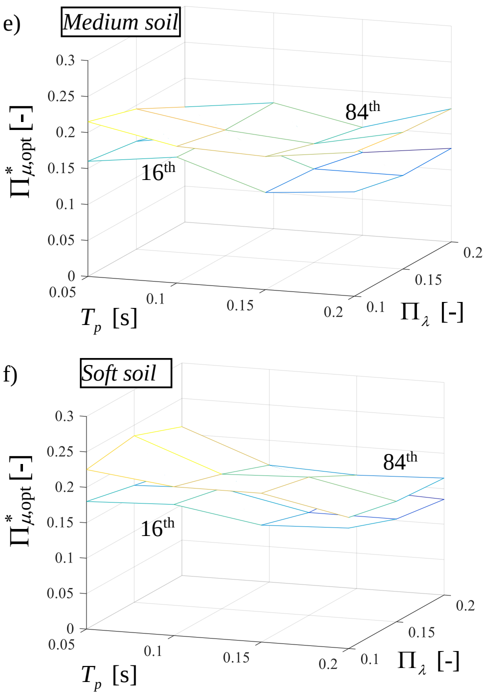




Figure 12 . Optimal values of normalized friction that minimize the 84th and 16th percentiles of the normalized pier displacements vs. $\Pi_\lambda$ and $T_p$ for each soil type and for $T_d$ =2s (a,b,c) and $T_d$ =4s (d,e,f).

As previously discussed, it is also possible to observe that higher values of the optimum friction coefficient are required, especially for low isolated periods, for soft soil condition in order to reduce the bearing displacements and, consequently, the forces transmitted to the pier as well as to increase the energy dissipation (equivalent damping). A reversal of this trend occurs for high values of both the isolation period and pier period, when it is necessary to dissipate more seismic energy input due to the resonance effect that affects the pier for stiff soil condition. In order to assure a high safety level, it might be of interest to define the values of $\Pi^*_{\mu,\mathrm{opt}}$ that minimize others response percentiles (Ryan and Chopra 2004). Figure 12 shows the optimal values of normalized friction that minimize the 84th and 16th percentiles of the normalized pier response for the different values of $\Pi_\lambda$, $T_p$, $T_d$ = 2s (Fig. 12a,b,c), $T_d$ = 4s (Fig. 12d,e,f) and for the three soil conditions. The trend is similar to the case of the 50th percentile. Regression expressions as statistics equations (Garzillo et al. 2015), (Golzio et al. 2013) to estimate the optimal friction coefficient may be found in (Castaldo et al. 2018c).

# 6 CONCLUSIONS

This paper describes the seismic performance of elastic bridge pier equipped with friction pendulum system (FPS) bearings in order to define the optimal isolator friction properties as a function of the structural properties and of the soil characteristics in terms of frequency content, corresponding to stiff, medium and soft soils, respectively. Assuming an equivalent two-degree-of-freedom model, representative, respectively, of the dynamic behaviour of a single-column bent viaduct, describing a continuous and infinitely rigid deck with an elastic pier, and the velocity-dependent FPS isolator behaviour, a non-dimensionalization of the motion equations is herein proposed. For each soil type, the uncertainty in the seismic inputs is taken into account by means of a set of 100 artificial non-stationary stochastic records, obtained through the power spectral density method, with different frequency content. By means of the proposed non-dimensionalization, a wide parametric analysis is developed for several isolator and pier properties, and for different soil conditions, by monitoring the response parameters of interest.

With reference to the deck response, the geometric mean of the normalized deck displacement increases slightly for increasing isolation period because of period elongation and it decreases significantly as normalized friction increases while it is not heavily influenced by the mass ratio. The dispersion for high isolation period increases for increasing values of normalized friction. The mass ratio does not affect significantly the response dispersion, especially for high pier periods. There are resonance effects for soft soil condition and low normalized friction values, and for stiff soil condition and high normalized friction values, particularly, for higher values of the pier period.

With reference to the pier response, the geometric mean of the normalized displacement decreases for increasing values of isolation period and of mass ratio as well as for decreasing values of pier period, whereas it first decreases and then increases for increasing values of normalized friction. Thus, there exists an optimal value of normalized friction coefficient such that the pier displacement is minimized for each soil condition. This optimal value varies in the range between 0.1 and 0.3 depending on the system parameters and the soil type. The values of the dispersion are generally very low. The other system parameters have a reduced influence on the dispersion compared to the influence of the normalized friction. There are resonance effects for the stiff soil condition with increasing normalized friction values, particularly, for higher values of pier period and lower isolation period values.

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