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Long-term dynamic monitoring and seismic response of the Milan Cathedral

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ABSTRACT
The Milan Cathedral, built between 1386 and 1813, is one of the largest masonry monuments ever built. Within the traditional collaboration between Politecnico di Milano and *Veneranda Fabbrica del Duomo di Milano* – the historic Institution established in 1387 and responsible for the preservation and development of the cathedral – a structural monitoring system was recently designed and implemented with the two-fold objective of assisting the condition-based structural maintenance of the cathedral and creating a large archive of experimental data useful to improve the knowledge of the monument.
The new monitoring system, fully computer based and with efficient transmission of the collected data, includes quasi-static and dynamic measurements. The dynamic monitoring is performed through seismometers (electro-dynamic velocity sensors) installed at the top of 14 selected piers and at 3 levels of the main spire.
After a concise historic background on the Milan Cathedral and the description of the dynamic monitoring system installed in the cathedral, the paper focuses on the results obtained during the first months of monitoring and the lessons learned in view of the Structural Health Monitoring (SHM) of the monument. Furthermore, as a far-field seismic event was recorded on January 15th, 2019 some comments are given on the church seismic response.

1 INTRODUCTION

The Institution named *Veneranda Fabbrica del Duomo di Milano* (Ferrari da Passano 1973), denoted as *Fabbrica* in the following, was established in 1387 to manage all operational aspects related to the construction, maintenance and restoration of the Milan Cathedral (Veneranda Fabbrica del Duomo 1865). After the completion of the church structures, the *Fabbrica* main mission moved from construction to maintenance, implying continuous inspection and architectural restoration of surfaces, decorations and statues in Candoglia marble. Furthermore, remarkable structural interventions are reported in the recent history of the cathedral, especially on the main spire (Nava 1845, Zacchi 1941) and the piers (Ferrari da Passano 1988) supporting the dome and the *tiburio* (i.e., the prismatic structure with octagonal base, which is built around the dome). Consequently, the preservation of the church has been traditionally assisted by the installation of different sensing devices, aimed at investigating specific issues.

During the recent assessment of the metallic tie-rods of the Milan Cathedral (Gentile et al. 2019) and taking profit of a joint research project between the *Fabbrica* and Politecnico di Milano, the idea to perform condition-based structural maintenance has been taking shape and a monitoring system was designed (Canali and Gentile 2018), with the objectives of providing the information needed for the condition-based structural maintenance and the creation of a large archive of experimental data useful to improve the knowledge of the monument.

The study of several historic documents, as well as the analysis of more recent information, allowed to identify the sub-structures to be specifically considered in the monitoring: (a) the main spire; (b) selected piers, which are placed in key areas of the church (i.e., apse, *tiburio*, transepts, main naves and façade); (c) the tie-rods subjected to high tensile stress or affected by slight damage (Gentile et al. 2019). The designed monitoring system includes a relatively large number of sensors for measuring both static (15 bi-axial tilt-meters and 12 vibrating wire extensometers) and dynamic parameters (36 uni-axial seismometers). In addition, the indoor and outdoor environmental parameters (temperature and humidity) are extensively monitored with the two-fold objective of establishing correlations with the structural parameters changes (see e.g. Gentile et al. 2016, Masciotta et al. 2016, Ubertini et al. 2017, Elyamani et al. 2017, Azzara et al. 2018) and evaluating the risks for the conservation of the main artifacts present in the cathedral (Aste et al. 2018).

The present paper is mainly aimed at describing the dynamic monitoring system installed in the Milan Cathedral and selected results obtained during the first 6 month of monitoring. After a concise historic background of the Milan Cathedral and a brief description of the dynamic monitoring hardware, the modal behaviour of the cathedral, identified in the first hours of monitoring, are presented and discussed. Subsequently, the evolution in time of the environmental and modal parameters during the first months of monitoring is addressed and some preliminary comments regarding the time-invariance of mode shapes and mode complexity are given within a SHM perspective (Cabboi et al. 2014). Finally, some comments are given on the effects of a far-field seismic event, recorded on January 15th, 2019.

2 THE MILAN CATHEDRAL

The Milan Cathedral (Figs. 1-3), built between 1386 and 1813, is the most representative landmark of the city of Milan and one of the largest masonry monuments ever built. The church exhibits a unique style of architecture, which is characterized by a fusion of European Gothic style and Lombardy tradition, with the presence of neo-classic, neo-gothic and even renaissance influences due to the long period required by the construction works.

An essential chronology of the construction phases is summarized in Fig. 2 with reference to the longitudinal section of the cathedral. It is further noticed that the last gate of the church was inaugurated on January 6th, 1965 and this is usually considered to be the end of monument construction.

The building spreads over an area of more than 10400 m^2, with a volume of about 300000 m^3, so that it is the second largest gothic cathedral in the world by volume and area. Moreover, the church exhibits the tallest main nave among gothic cathedrals, with the height of the vault intrados of the main nave being at about 45 m from the ground.

The perimeter load-bearing walls are built in dry masonry of varying thickness, with coatings in pink-veined white marble from the Candoglia quarries. Moreover, all the spires linked to the flying buttresses (Fig. 1) and the statues (about 3400) adorning the Cathedral are made in Candoglia marble.

Figure 1. Aerial view of the Milan Cathedral (courtesy of *Veneranda Fabbrica del Duomo di Milano*).

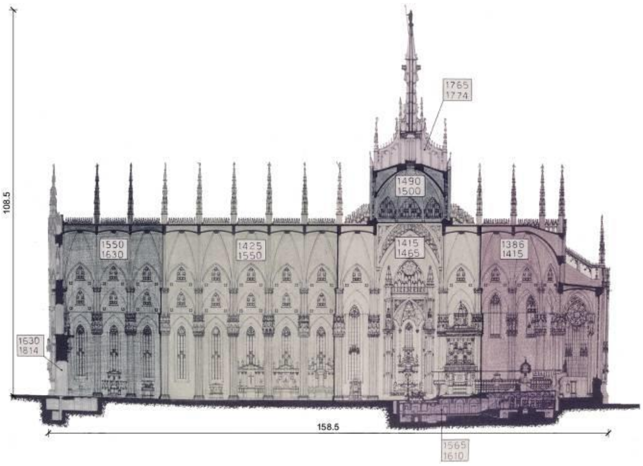

Figure 2. Longitudinal section of the Milan Cathedral (dimensions in m) and chronology of the main construction phases (courtesy of *Veneranda Fabbrica del Duomo di Milano*).

Figure 3. Milan Cathedral: inside view of arches, vaults and iron tie-rods.

When compared with other gothic cathedrals, the Milan Cathedral exhibits a peculiar structural system, with metallic tie-rods being permanently installed under each vault (Fig. 3) and designed to exert an active part in resisting the lateral thrusts. Historical documents (Veneranda Fabbrica del Duomo 1865) testify that the tension bars in the Milan Cathedral were permanently installed on the top of the piers during the construction with the aim of reducing the horizontal thrust on the lateral buttresses, as those buttresses were judged too slender by the French architect Jean Mignot. A total of 122 metallic tie-rods (Fig. 3) is nowadays present in the cathedral and most of them are the original elements dating back to the age of construction.

The overall dimensions of the Latin cross-shaped plan are about 66 m × 158 m (Fig. 4), with the aisles and the central naves spanning 9.6 m and 19.2 m, respectively.

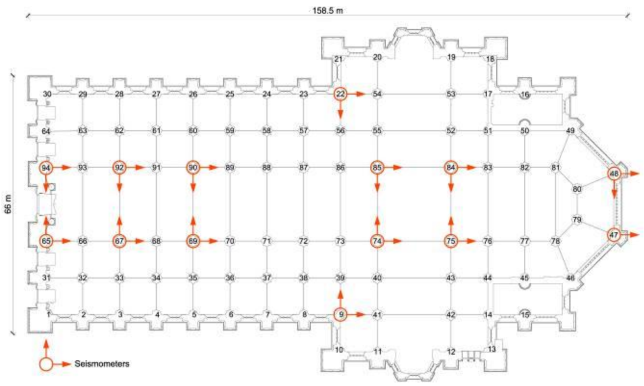

Figure 4. Plan of the Milan Cathedral (dimensions in m) and general layout of the seismometers installed in the church.

3 THE DYNAMIC MONITORING SYSTEM

In order to characterize the current health state of the Milan Cathedral and to address its condition-based structural maintenance, the following key parameters are continuously measured or identified from the measurements: (a) bi-axial tilt of the capital of selected piers (i.e., piers 31, 64, 69, 90, 11, 20, 74-75, 84-85 and 47-48 in Fig. 4) and main spire; (b) strain in tie-rods 28-62, 27-61, 26-60, 35-69, 25-59, 7-37, 9-39, 39-40, 56-55, 45-77, 38-72 and 87-57 (Fig. 4); (c) temperature and humidity on the same capitals where the tilt is measured; (d) resonant frequencies, mode shapes and damping ratios.

Due to the different technical characteristics of the various sensing devices and sampling rates of the data acquisition, two separated long-term monitoring systems – one static and the other dynamic – were installed in the church. Strains, rotations and environmental data were collected at a rate of two samples per hour, whereas sampling frequency of 100 Hz was adopted in the dynamic monitoring. It is further noticed that the monitoring systems are fully computer based and their architecture has been established in order to minimize wiring, as well as the visual impact of the sensing devices inside the church.

The dynamic monitoring system installed in the Cathedral of Milan is entirely based on SARA SS45 seismometers (electro-dynamic velocity transducers). The seismometer choice is motivated by: (a) the high sensitivity (78 V/[m/s]) and the excellent performance of electro-dynamic transducers in the low frequency range ($f \leq 100$ Hz); (b) the un-necessity of powering the sensors; (c) the possibility of obtaining a good estimate of the displacement time series by integrating the velocity records.

The dynamic monitoring system consists of:

- 13 bi-axial seismometers and 1 mono-axial seismometer, installed at the top of selected piers inside the Cathedral (Fig. 4) and measuring the velocity in the two orthogonal N-S (transversal) and E-W (longitudinal) directions. The sensors installed on piers (94, 92, 90), (65, 67, 69), (22, 85, 84), (9, 74, 75) and (47, 48) are grouped and wired to five 24-bit digitizers SARA SL06; each digitizer is equipped with A/D conversion system, 8 GB memory, synchronization by GPS, back-up battery and UMTS modem for data transfer;
- 3×3 mono-axial seismometers, installed at the same levels of the main spire hosting the biaxial tilt-meters belonging to the static monitoring. The three sensors installed at each level are wired to 24-bit digitizer, with the digitizers being connected to a switch for data transfer.

The velocity data are transferred in real time to the *Fabbrica* workstation and stored in separate files (compressed mini-seed format) of 1 hour. Every hour, the automatic signal processing involves the following tasks: (a) pre-processing the raw data (to compensate the low-frequency attenuation of the sensor) using the SEISMOWIN commercial software (https://www.agisco.it/en/) and subsequent saving of the time series in text format; (b) data analysis to extract the maximum and the root mean square values and creation of a file in Matlab (.mat) format; (c) low-pass filtering and down-sampling (to reduce the sampling frequency from 100 Hz to 20 Hz), and creation of a database of files (in binary or text format) for the application of the modal identification tools; (d) automated modal estimation and tracking.

The modal parameters of the cathedral and of the main spire are independently identified from 3600 s velocity datasets using a fully automated procedure, based on the covariance-driven Stochastic Subspace Identification (SSI-Cov) algorithm and developed in (Cabboi et al. 2017).

4 DYNAMIC CHARACTERISTICS OF THE CATHEDRAL AND SHM STRATEGY

The grid of seismometers permanently installed in the cathedral allowed the identification and fairly good spatial description of a relatively large number of key vibration modes. Typical results obtained in the first day of continuous monitoring, in terms of resonant frequencies and mode shapes, are shown in Figs. 5-6.

Fig. 5 exemplifies the stabilization diagram obtained by applying the automated modal parameter estimation algorithm (Cabboi et al. 2017) to the dataset (27 channels of data, Fig. 4) recorded on 17/10/2018 (h 12:00-13:00). The stabilization diagram in Fig 5 is shown after cleaning and selection (clustering) of physical modes, along with the first Singular Value (SV) line of the spectral matrix, which is the mode indication function adopted in the Frequency Domain Decomposition (FDD) [32].

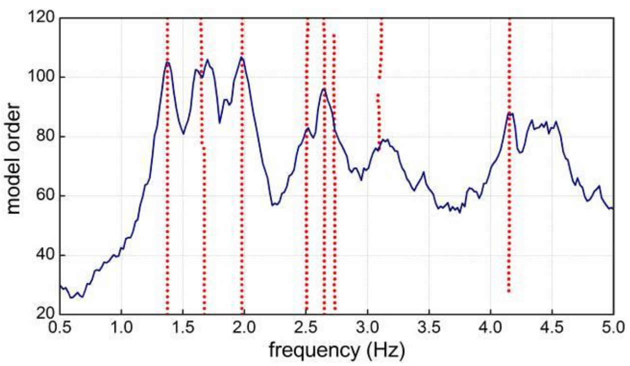

Figure 5. Typical stabilization diagram obtained from the signals collected in the Milan Cathedral (27 channels of velocity data, 17/10/2018, h 12:00-13:00).

The inspection of Fig. 5 highlights eight alignments of the stable poles, providing a clear indication of the structural modes and those alignments of poles generally correspond to local maxima in the first SV line. The mode shapes corresponding to the lower 4 modes are shown in Fig. 6 (where the blue colour refers to modes with dominant motion in the N-S direction and the red colour refers to modes with dominant deflection in the E-W direction). The inspection of Fig. 6 allows the following comments: (a) the lower two modes, as expected, are global sway modes of the cathedral along the transverse (N-S, 1.39 Hz, Fig. 6a) and longitudinal (E-W, 1.70 Hz, Fig. 6b) direction; (b) the subsequent two modes (1.96 and 2.48 Hz, Figs. 6c and 6d) involve dominant motion in the N-S direction and bending of the

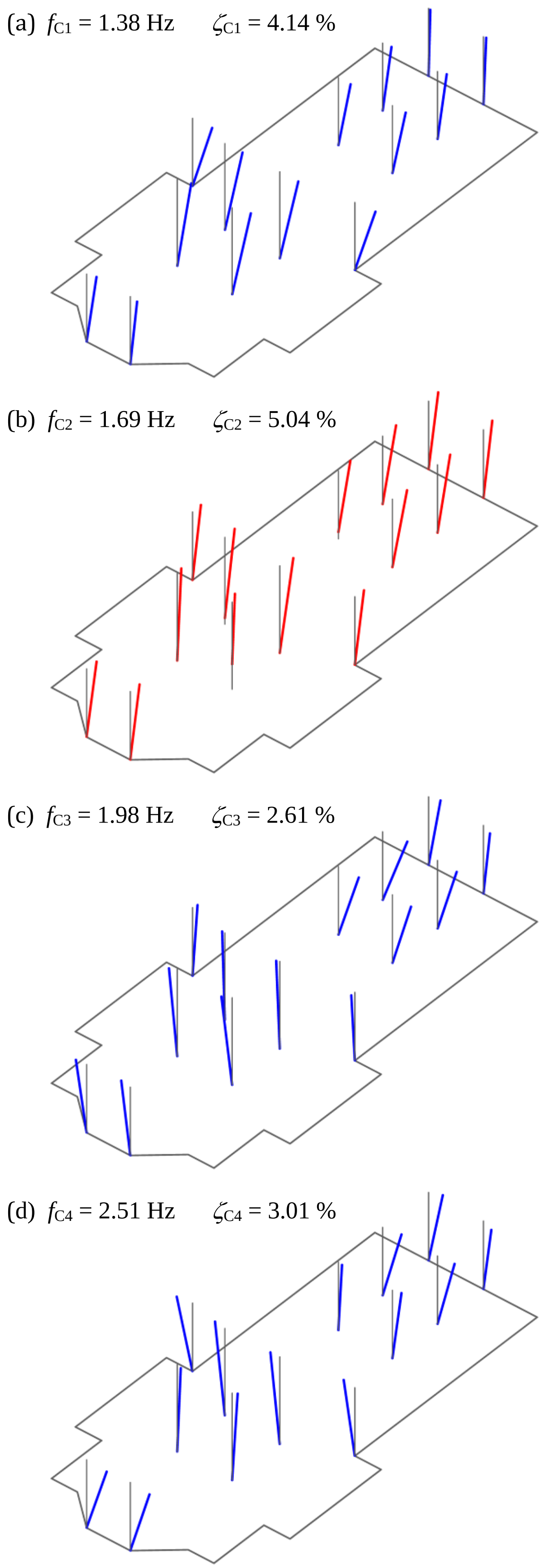

Figure 6. Selected vibration modes of the Milan Cathedral (17/10/2019, h 12:00-13:00).

naves; (c) the damping ratios of all modes are quite high. The higher modes (not shown in Fig. 6) are characterized by more complex mode shapes: for example, the 5th mode (2.64 Hz) involves out-of-phase bending of the North and South naves, whereas the 6th mode (2.76 Hz) involves out-of-phase motion of the façade and apse in the E-W direction.

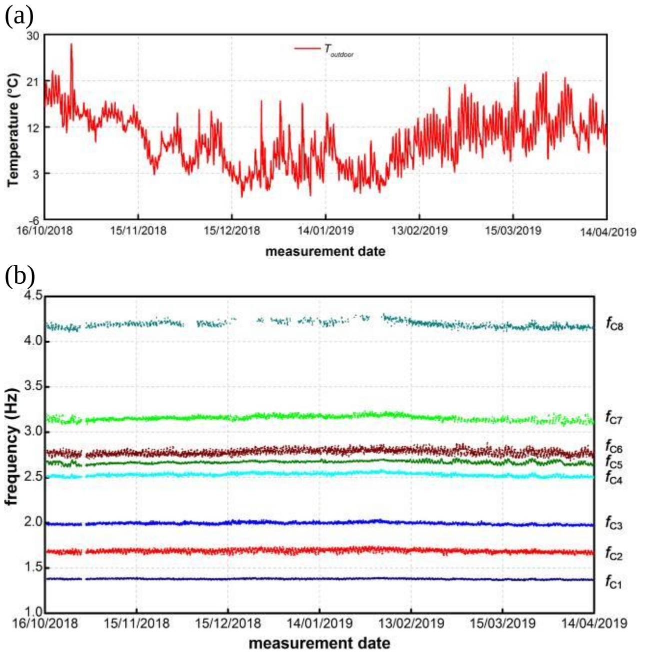

Figure 7. Time evolution (from 16/10/2018 to 13/04/2019) of: (a) the outdoor temperature and (b) the automatically identified natural frequencies.

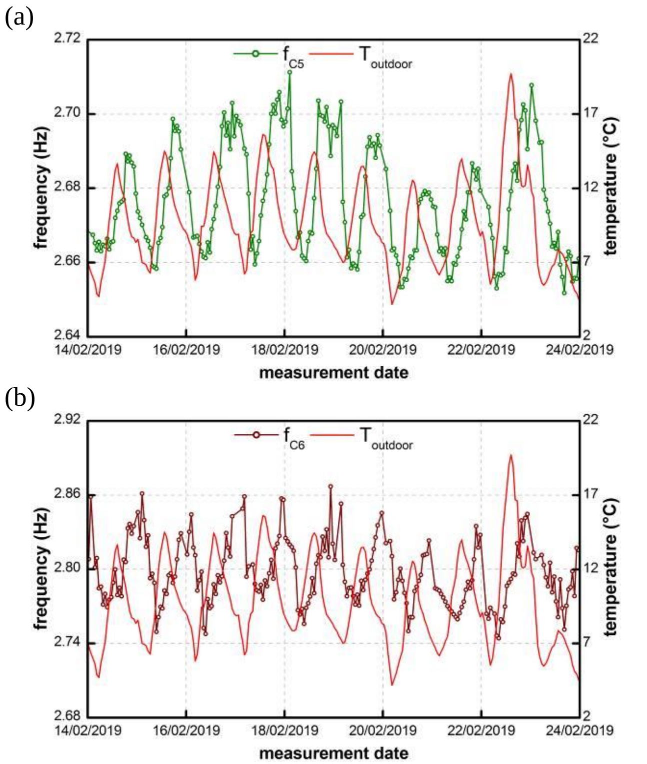

Figure 8. Zoomed variation of outdoor temperature and identified natural frequency: (a) mode C5; (b) mode C6.

Table 1. Statistics of the natural frequencies automatically identified from 16/10/2018 to 13/04/2019.

Mode	f_{ave} (Hz)	σ_f (Hz)	f_{min} (Hz)	f_{max} (Hz)
C1	1.378	0.006	1.359	1.394
C2	1.687	0.018	1.630	1.739
C3	1.991	0.013	1.956	2.036
C4	2.530	0.018	2.484	2.586
C5	2.666	0.015	2.617	2.711
C6	2.779	0.029	2.683	2.883
C7	3.156	0.025	3.066	3.234
C8	4.185	0.033	4.107	4.307

Fig. 7a shows the evolution in time of the outdoor air temperature during the first 6 months of monitoring. The corresponding variation of the modal frequencies versus time is illustrated in Fig. 7b, whereas the statistics of frequencies are listed in Table 1. Table 1 summarizes the mean value (f_{ave}) and the standard deviation (σ_f) of the frequency estimates as well as the extreme values (f_{min}, f_{max}) of each frequency. It should be noticed that the lower modes have been automatically identified with high occurrence during the first 6 months, with the identification rate being larger than 98% for modes C1-C5 and equal to 90.7% for mode C6; on the other hand, the identification rate decreases to 77.1% for modes C7 and to 36% for mode C8.

As shown in Table 1 and Fig. 7b, all identified resonant frequencies exhibit slight but clear variations in time: the standard deviation ranges between 0.06 Hz (mode C1) and 0.34 Hz (mode C8) and the frequency fluctuations are almost completely driven by the changing temperature, whereas the average relative humidity seems to slightly affects only the frequency of mode C1. Especially the frequencies associated to modes with dominant motion in the longitudinal direction of the Cathedral (modes C2 and C6) reveal clear daily variations (Fig. 7b), with the natural frequencies increasing as (indoor and outdoor) temperature decreases. This peculiar trend is further exemplified in Fig. 8, where the frequency and outdoor temperature variation are zoomed, in a time interval of 10 days, for modes C5 (Fig. 8a) and C6 (Fig. 8b): the peaks in the modal frequency correspond to the valleys (relative minima) of the temperature.

The negative dependence of frequencies on temperature is a distinctive behaviour of the Milan Cathedral, with this trend being very different from what reported in almost all the long-term studies on masonry structures, either towers (see e.g. Gentile et al. 2016, Ubertini et al. 2017, Azzara et al. 2018) or churches (Masciotta et al. 2016, Elyamani et al. 2017). In those studies, the natural frequencies increase with increased temperature, with this behaviour being explained as the effect of the closure of cracks, minor discontinuities and mortar gaps induced by the thermal expansion of materials, so that a temporary stiffening of the structure is generated.

In the authors' opinion, the negative frequency-temperature correlation in the Milan Cathedral is determined by the structural arrangement, consisting of double vault system constrained by an extended net of metallic tie-rods (Fig. 3): as a matter of fact, the extensometers installed in the tie-rods highlight a generalized strain increase with decreased temperature, so that the corresponding increased forces in the tension bars conceivably exert a stiffening action on the overall structure. It is worth mentioning that a similar negative frequency-temperature correlation is reported in the literature only for a monumental building (Kita et al. 2019), which is characterized by the presence of metallic tie-rods, as well.

As pointed out in section 3, a relatively large number (27) of seismometers is installed in the cathedral, so that the investigation of mode shape variations (see e.g. Cabboi et al. 2014) is allowed as well: this analysis is of utmost importance because mode shapes provide both local and global information on the structure. In addition, even for complex systems, the mode shapes are supposed to be less sensitive than natural frequencies to environmental changes (which affect almost uniformly the structure) and conceivably more sensitive to local structural changes.

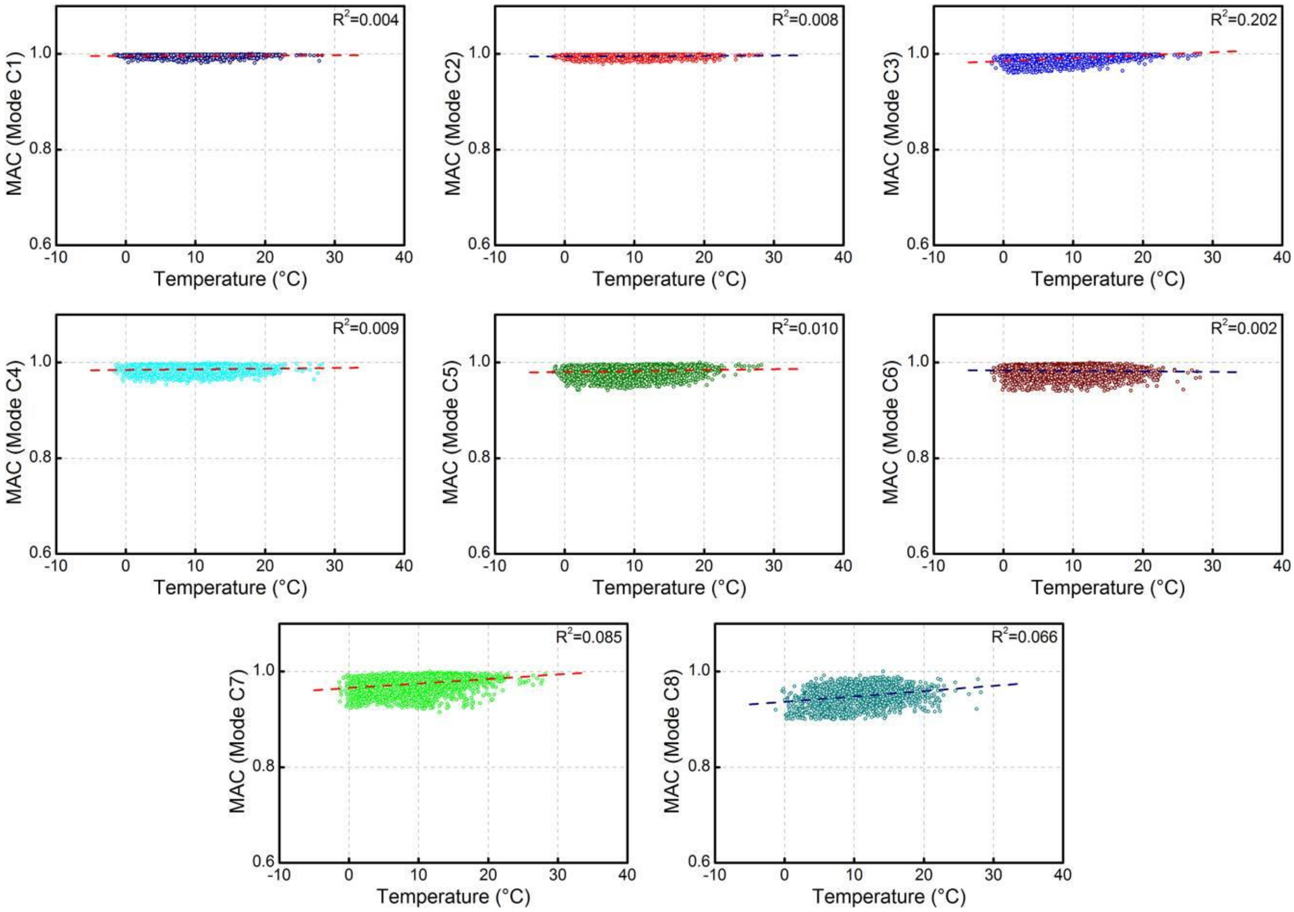

Figure 9. Correlation between the outdoor temperature and the MAC of identified modes (from 16/10/2018 to 13/04/2019).

The correlation between outdoor temperature and mode shape variation, in terms of the MAC, is shown in Fig. 9. The inspection of the figure confirms that the MAC values (i.e. the mode shapes) are approximately independent from temperature and time invariant, even if the standard deviation of the MAC tend to increase with the increasing mode order (and the increasing spatial complexity of the mode shape).

Table 2 summarizes the statistical characteristics of the MAC through the average value (MAC_{ave}), standard deviation (σ_{MAC}) and minimum value (MAC_{min}). In addition, the coefficient of determination $R^2_{MAC\text{-}T}$ is also shown in Table 2 (as well as in Fig. 9), in order to underline that the outdoor temperature practically does not affect the MAC of modes C1-C2 and C4-C6 ($R^2 < 0.010$) and has a minor influence on the MAC of the other modes (with the coefficient of determination ranging between 0.066 and 0.202). It is further noticed that a similar time invariance has been found also for the indices describing the mode complexity (see e.g. Pappa et al. 1992).

Consequently, the SHM of the historic monument will be performed not only using the resonant frequencies as novelty sensitive features but also checking the invariance of the mode shapes (Fig. 9) and related modal complexity.

It is further noticed that the availability of quasi-static measurement of structural responses (such as the tilt of selected piers and the strain on a certain number of tie-rods) might contribute – directly or through the fusion with the data coming from OMA-based monitoring – to check the structural health condition of the cathedral.

Table 2. Statistical description of the MAC values from 16/10/2018 to 13/04/2019.

Mode	MAC_{ave}	σ_{MAC}	MAC_{min}	R^2_{MAC-T}
C1	0.997	0.002	0.981	0.004
C2	0.995	0.003	0.979	0.008
C3	0.991	0.007	0.960	0.202
C4	0.986	0.007	0.954	0.009
C5	0.982	0.010	0.942	0.010
C6	0.982	0.011	0.940	0.002
C7	0.974	0.016	0.915	0.085
C8	0.949	0.020	0.901	0.066

5 RESPONSE TO A FAR-FIELD SEISMIC EVENT

After the beginning of the continuous dynamic monitoring, the response to a far-field seismic event was clearly measured on January 15th 2019. The earthquake response was recorded just after midnight and corresponded to a seismic event of 4.6 Richter magnitude with epicentre in Ravenna.

Fig. 10 exemplifies the velocity time history measured in the N-S direction on the capital of pier 9 (Fig. 4), belonging to the transept. It should be noticed that: (a) the maximum velocity (of about 0.42 mm/s) measured during the earthquake exceeds of 4-8 times the maximum response recorded in operational conditions; (b) the maximum response of the cathedral was measured in the N-S (transverse) direction and

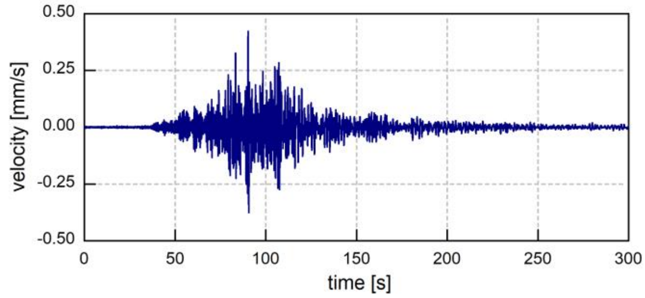

Figure 10. N-S velocity measured on the capital of pier 9 during the seismic event of January 15th, 2019.

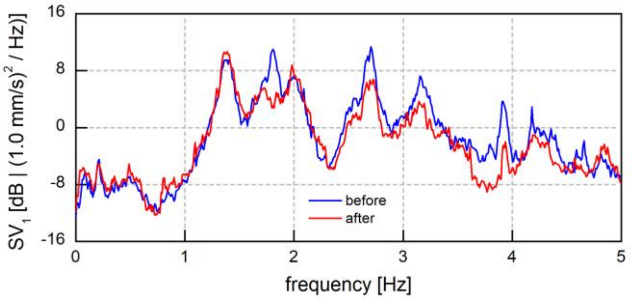

Figure 11. Comparison between the first SV lines corresponding to datasets collected before and after the seismic event of January 15th, 2019.

along the central alignment of columns including the instrumented piers 9, 74, 85 and 22 (belonging to the transept and *tiburio* region, respectively).

In order to quickly verify that the earthquake did not induce appreciable structural effects on the monument, time windows of 3000 s − which were recorded before and after the seismic event − have been considered. Fig. 11 shows the comparison between the first SV lines of the spectral matrix of the two datasets; the two SV lines are indeed very similar and especially the frequency of the relative maxima (peaks) corresponding to the N-S modes turns out to be practically unchanged. In other words, the earthquake did not induce any frequency shifts. Of course, a more refined analysis was performed by using the automatically identified resonant frequencies and mode shapes and this analysis confirms the evidence of no variation in terms of modal parameters (resonant frequencies, mode shapes and modal complexity).

6 CONCLUSIONS

The paper focuses on the description of the monitoring system recently designed and installed in the Milan Cathedral. The monitoring system, aimed at enhancing the knowledge of the historic building and assisting its condition-based structural maintenance, includes more than 120 sensors belonging to different classes (i.e., a network of 12 extensometers, 15 bi-axial tilt-meters, 36 seismometers, 30 temperature sensors, 12 hygrometers and 1 weather station) and is fully computer based and characterized by distributed architecture.

Special emphasis is given on the dynamic measurements, the automated extraction of modal signatures and the influence of environmental parameters on the variations observed in the dynamic characteristics of the cathedral. Based on the results obtained during the first 6 months of continuous monitoring (i.e., between 16/10/2018 and 13/04/2019), the following main conclusions can be drawn:

1. The application of state-of-art tools for automated operational modal analysis allows estimate and tracking of eight resonant frequencies in the frequency interval 0-5 Hz. The automatically identified frequencies involve dominant motion in the main transverse (N-S) and longitudinal (E-W) direction of the Cathedral;
2. The (outdoor and indoor) temperature turned out to be a dominant driver of the variations observed in the natural frequencies of all modes. The relative humidity seems to slightly affects only the frequency of the first mode;
3. Even if the observation period is relatively limited, the frequency-temperature correlations reveal a distinctive trend, which is very different from what reported in almost all the long-term studies on historic masonry structures. All resonant frequencies of the Milan Cathedral almost linearly increase with decreased temperature, and the negative dependence of modal frequencies on temperature is conceivably determined by the action exerted by the metallic tie-rods in the structural arrangement of the monumental building;
4. The mode shapes of the cathedral and the corresponding mode complexity do not exhibit appreciable fluctuations associated to the environmental effects, so that an appropriate strategy of SHM should be based on the time invariance of those parameters.

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